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A Method for Quantifying Vehicle Crush Stiffness Coefficients

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ABSTRACT

The accuracy of an accident reconstruction, which employs the damage analysis feature of the CRASH3 computer program, is directly related to the accuracy of the crush stiffness coefficients employed. Crush stiffness coefficients, however, are available only through a limited number of publications and for a limited number of vehicles. In addition, assumptions made in the determination of these published stiffness coefficients bring their accuracy into question and, as a result, limit their value to a reconstructing engineer. It is concluded, therefore, that an engineer must use a critical eye when viewing the results of a CRASH3 reconstruction in which these stiffness coefficients were employed.

A method is set forth for quantifying stiffness coefficients from crash test data available in a database which can be obtained from the National Highway Traffic Safety Administration (NHTSA). This method will allow reconstructing engineers to have a greater level of confidence in the results of an accident reconstruction in which the damage analysis feature of CRASH3 is employed.

INTRODUCTION

The CRASH3 computer program is increasingly being used as a tool by engineers to reconstruct automobile accidents. The damage analysis portion of CRASH3 provides a means to evaluate the energy expended during the collision phase of an automobile accident. This crush energy can then be used in a separate conservation of energy analysis to reconstruct an automobile accident with greater confidence than in the past when CRASH3 was not available. The CRASH3 damage analysis also provides an engineer with a method for quantifying the momentum exchange associated with a collision through the use of a delta-V.

The accuracy of the CRASH3 damage analysis, however, is dependent on the accuracy of the crush stiffness coefficients input into the program. The saying "garbage in, garbage out" applies to all computer programs and CRASH3 is not an exception. A comparison of the crush energy associated with a certain crush depth indicates that a high degree of variation exists in the stiffness characteristics of different vehicles *[1]. An inaccuracy of 10% in the stiffness coefficients can result in an inaccuracy in excess of 6% in the determination of delta-V [2]. As a result of this potential inaccuracy, a reconstructing engineer should associate a confidence level with the results of the damage analysis which reflects the accuracy of the stiffness coefficients which were used in CRASH3. Confidence levels which reflect a high potential for inaccuracy, however, degrade an engineer's ability to render a meaningful opinion.

The limited availability and accuracy of stiffness coefficient data is becoming a problem of increasing importance as more

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*The numbers in the brackets refer to references listed at the end of the paper.

engineers use CRASH3 as a tool in accident reconstruction. Crush stiffness data is available from two sources, both of which have deficiencies. The first is the CRASH3 User's Guide and Technical Manual which contains a table of stiffness coefficients [3]. Vehicles are grouped in this table into 8 categories with each category assigned a single set of stiffness coefficients. The use of these values assumes that similar sized vehicles have similar stiffness characteristics. This assumption is clearly contrary to the aforementioned high degree of variance in stiffness characteristics. The crush energy determined with these values, therefore, should be considered to have a high probability for significant inaccuracy.

A second source of stiffness coefficient data is Engineering Dynamics Corporation (EDC). EDC published a reference manual in 1987 which contains 590 sets of stiffness coefficients for vehicles which were built before 1985 [4]. The calculation method used to determine the stiffness coefficients for angled Frontal Fixed Barrier (FFB) collisions did not take into account the manner in which CRASH3 adjusts the crush energy for a non-perpendicular Principle Direction of Force (PDOF). This resulted in the determination of high stiffness coefficients for these vehicles. When these high stiffness coefficients are used in a reconstruction involving a non-perpendicular PDOF, CRASH3 adjusts the crush energy for the non-perpendicular PDOF and, as a result, the inaccuracy in the stiffness coefficients is magnified. This could result in the determination of an extremely high crush energy.

The calculation method employed by EDC also used an average value for the crush depths of each vehicle. The use of an average crush depth assumes that the crush profile is uniform. This assumption reduces the accuracy of the calculated stiffness coefficients to the extent that the crush profile was not uniform. This reduction in accuracy is due to crush energy being proportional to the square of the crush depth. An engineer using stiffness coefficients from the EDC reference manual should review the corresponding original crash test data to determine an appropriate level of confidence in the accuracy of the stiffness coefficients. Applying stiffness coefficients from this source to similar vehicles built after 1984 also should be accompanied by a determination of an appropriate confidence level.

The objective of this paper is to provide engineers with an alternative to using these published stiffness coefficients. A method is set forth which will allow an engineer to determine stiffness coefficients from the crash test data which can be obtained from the National Highway Traffic Safety Administration (NHTSA). The Vehicle Crash Test Data Base (VCTDB), which is a compilation of reports on crash tests performed under contract for the Office of Vehicle Research, can be obtained from the NHTSA on magnetic tape [5]. Individual reports also can be obtained from the NHTSA on microfiche.

The VCTDB contains crash test data for in excess of 1000 post-1979 model year vehicles. The following crash test configurations are included in the VCTDB:

1. Frontal Fixed Barrier (FFB) Collisions

2. Angled FFB Collisions

3. Frontal/Rear Movable Barrier Collisions
   -Deformable Barriers (F/RMDB) and
   Non-deformable Barriers (F/RMNB)

4. Side Movable Deformable Barrier (SMDB) Collisions

This paper will address a method for quantifying stiffness coefficients for FFB collisions, angled FFB collisions and F/RMNB collisions. The method for determining the stiffness coefficients for collisions with deformable barriers is complicated by the need to determine the energy expended in the crushing of the deformable barrier. The SMDB collisions are further complicated by the need to determine the post collision rotational energy of the test vehicle and the barrier. The later test configurations can be addressed in a future paper.

DISCUSSION

History

Campbell presented data which defined a linear relationship between the collision speed of an automobile and the magnitude of the resulting crush in FFB collisions [6].

This relationship was described by the linear equation:

$$V = b_0 + b_1 C \tag{1}$$

Campbell in determining the slope, b_1, extrapolated the slope to zero crush. This resulted in intercepts, b_0, which ranged from 3.0 to 6.85 mph. These values are representative of the 1971-1974 model year domestic vehicles used in the analysis.

Campbell used this linear relationship as a basis for a model of the force-deflection characteristics of the front end of an automobile. He assumed that the force exerted on an automobile per unit width of crush, is linearly proportional to the crush depth.

$$F = A + BC \qquad [\text{lb/in}] \tag{2}$$

where:

A= the stiffness coefficient which represents the maximum force per unit width of the contact area which produces no crush, [lb/in]

B= the stiffness coefficient which represents the ratio of the force per unit width of the contact area to the crush depth, [lb/in^2]

C= the crush depth, [in].

A comparison of the crush energy per unit weight of the test vehicles was used by Campbell to rate the severity of the collisions. He defined this energy as an Equivalent Barrier Speed (EBS).

$$Crush\ Energy\,(E) = \frac{W}{2g}(EBS)^2 \tag{3}$$

The crush energy also could be determined by integrating the force-deflection equation (2) over the crush depth, L, and the crush width, C. The energy absorbed by a vehicle which produces no residual crush was treated by Campbell as being a constant.

$$E = \int_0^L\int_0^C (A + BC)\, dC\, dL + Constant \tag{4}$$

By substituting equation (1) for the EBS term in equation (3), equating this with equation (4) and integrating over a uniform crush profile, Campbell demonstrated that the stiffness coefficients, A and B, could be expressed as functions of b_0 and b_1.

$$A = \frac{W b_0 b_1}{gL} \tag{5}$$

$$B = \frac{W b_1^2}{gL} \tag{6}$$

$$Constant = \frac{W}{2g} b_0^2 \tag{7}$$

Equations (5), (6) and (7) were then substituted back into equations (3) and (4).

$$\frac{W(EBS)^2}{2g} = \frac{W}{gL}\int_0^L\int_0^C (b_0 b_1 + b_1^2 C)\, dC\, dL + \frac{W b_0^2}{2g} \tag{8}$$

Campbell solved for the EBS by integrating equation (8) over a non-uniform crush profile where the crush profile was described by two crush depths, C_1 and C_2. This resulted in the EBS being expressed as a function of b_0 and b_1.

The CRASH3 algorithm is based on Campbell's work and likewise assumes that a linear force-deflection model adequately describes the structural characteristics of the front end of vehicles [3]. The CRASH3 algorithm, however, extends this assumption to include the side and rear end of vehicles. The crush profile in the CRASH3 algorithm also is defined by up to 6 equally spaced crush depths.

Two major differences exist between Campbell's EBS and the CRASH3 algorithm. CRASH3 treats the absorbed energy that produces no residual crush as being proportional to the contact width rather than as a constant. Equation (7) is rewritten to reflect this change.

$$\frac{\frac{W}{2g} b_0^2}{L} = \frac{W b_0^2}{2gL} = \frac{A^2}{2B} \tag{9}$$

CRASH3 also models the collision phase with rigid-body kinetics. As a result, collision severity is expressed in terms of a delta-V which is the speed change experienced during the collision phase.

Equation (8) is rewritten in CRASH3 in terms of delta-V and the stiffness coefficients, A and B, and is integrated based upon a trapezoidal approximation of the crush profile defined by crush depths C_1 through C_6. The resulting equation is used by CRASH3 to describe the crush energy.

$$E=\frac{L}{5}\left(\frac{A\alpha}{2}+\frac{B\beta}{6}+\frac{5A^2}{2B}\right)=\frac{W}{2g}(\Delta V)^2 \qquad (10)$$

where: $\alpha = C_1+2(C_2+C_3+C_4+C_5)+C_6$

$$\beta = C_1^2+2(C_2^2+C_3^2+C_4^2+C_5^2)+C_6^2 + C_1C_2+C_2C_3+C_3C_4+C_4C_5+C_5C_6$$

$C_1 \rightarrow C_6 = crush\ depth,\ [in]$

METHOD

FFB Collisions

Equations (5) and (6) define the stiffness coefficients as a function of b_0 and b_1. If a reasonable approximation for b_0 of 5 mph is substituted into these equations, b_1 is the only term which can not be obtained directly from the test data included in the VCTDB*. The slope, b_1, can be determined by rewriting equation (1).

$$b_1=\frac{V-b_0}{C} \qquad (11)$$

This equation is applicable to FFB collision test data where the crush, C, is uniform. This is the method used by EDC in their reference manual [4]. For test data which meets this restriction, the stiffness coefficients can be quantified by substituting equation (11) into equations (5) and (6) and performing a units conversion.

$$A=\frac{0.802\ Wb_0(V-b_0)}{LC} \quad [\mathrm{lb/in}] \qquad (12)$$

$$B=\frac{0.802\ W(V-b_0)^2}{LC^2} \quad [\mathrm{lb/in^2}] \qquad (13)$$

*Analysis has been performed on crash test data obtained from the VCTDB [1]. This analysis indicates that 5 mph is a reasonable value for b_0 for FFB collisions.

where: b_0= FFB collision speed which coincides with the onset of crush, [mph]
C= uniform crush depth, [in]
L= crush profile width, [in]
W= vehicle weight, [lb]
V= FFB collision speed, [mph]

For FFB collisions where the crush profile is not uniform, equation (10) needs to be rewritten in terms of b_0 and b_1 and rearranged to solve for b_1 [mph/in].

$$b_1=-b_0\alpha+\frac{\sqrt{(b_0\alpha)^2-\frac{20\beta}{3}(b_0^2-V^2)}}{\frac{2}{3}\beta} \qquad (14)$$

Next a dimensional analysis applied to equations (5) and (6) yields:

$$A=\frac{0.802\ Wb_0b_1}{L} \quad [\mathrm{lb/in}] \qquad (15)$$

$$B=\frac{0.802\ Wb_1^2}{L} \quad [\mathrm{lb/in^2}] \qquad (16)$$

Finally the determined value for b_1 can be used in equations (15) and (16) to quantify the stiffness coefficients for FFB collisions involving non-uniform crush profiles.

Angled FFB Collisions

During angled FFB collisions the intervehicular force does not act along a line perpendicular to the surface of the involved side of the vehicle. The crush depth, however, is measured perpendicular to the surface of the vehicle. An adjustment, therefore, needs to be made to account for the additional distance over which the surface of the vehicle is displaced while being crushed along this non-perpendicular line of action. The CRASH3 program accounts for this additional distance by multiplying the crush energy by an energy correction factor.

$$E_{Corrected}=E\left(\frac{1}{\cos^2\theta}\right)=\frac{E}{\cos^2\theta} \qquad (17)$$

where:

Theta= the angle between the PDOF and a line perpendicular to the involved surface.

Rearranging equation (17) and solving for the uncorrected crush energy in terms of the collision speed yields:

$$E=\frac{W}{2g}(V)^2\cos^2\theta=\frac{W}{2g}(V\cos\theta)^2 \qquad (18)$$

This produces an effective speed term which accounts for the energy correction factor used in CRASH3.

$$V_{eff}=V\cos\theta \qquad (19)$$

Equation (14) then can be rewritten for angled FFB collisions. It should be noted that equation (11) can not be used due to the non-uniform crush profiles which result from angled FFB collisions.

$$b_1=\frac{-b_0\alpha+\sqrt{(b_0\alpha)^2-\frac{20\beta}{3}\left(b_0^2-V_{eff}^2\right)}}{\frac{2}{3}\beta} \qquad (20)$$

Equation (20) can now be used to solve for b_1 [mph/in]. This value then can be used in equations (15) and (16) to quantify the stiffness coefficients, A and B, for angled FFB collisions.

F/RMNB Collisions

The use of movable barriers require that an additional energy balance be performed. This energy balance can be used to produce an effective speed term , $V_{v,eff}$ [mph], which represents the energy expended in producing the vehicular crush [4].

$$V_{v,eff}=\sqrt{\frac{W_b}{W_v}\left(V_{b1}^2-V_{b2}^2\right)+\left(V_{v1}^2-V_{v2}^2\right)} \qquad (21)$$

where:

V_{b1}= barrier collision speed, [mph]
V_{b2}= barrier separation speed, [mph]
V_{v1}= vehicle collision speed, [mph]
V_{v2}= vehicle separation speed, [mph]
W_b = barrier weight, [lb]
W_v = vehicle weight, [lb]

After obtaining these values from the VCTDB, equation (21) can be solved. The value of $V_{v,eff}$ then can be used in a rewritten form of equation (14) to solve for b_1 [mph/in].

$$b_1=\frac{-b_0\alpha+\sqrt{(b_0\alpha)^2-\frac{20\beta}{3}\left(b_0^2-V_{v,eff}^2\right)}}{\frac{2}{3}\beta} \qquad (22)$$

The stiffness coefficients from a F/RMNB collision then can be quantified by applying the value of b_1 in equations (15) and (16).

SUMMARY

1. The need for accurate stiffness coefficients was identified. The limited availability and accuracy of published stiffness coefficients were discussed. A source for obtaining crash test data is available. The reconstructing engineer, however, must be able to convert the test data into stiffness coefficients.

2. A brief history of the evolution of the CRASH3 algorithm was discussed.

3. A method was set forth for quantifying the stiffness coefficients, A and B, for FFB collisions, angled FFB collisions and F/RMNB collisions.

4. When reconstructing an automobile accident, the use of the quantified stiffness coefficients should reduce the inaccuracy involved in determining the crush energy. The reconstructing engineer, however, still should use an appropriate confidence level which reflects the possible inaccuracy in measurements and in the assumed linear force-deflection characteristics used in the CRASH3 algorithm.

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APPENDIX

Derivation of the Slope Equation for Non-uniform Crush Profiles

For FFB collisions where the crush profile is not uniform, the CRASH3 energy equation (A-1) needs to be rewritten in terms of b_0 and b_1.

$$E=\frac{L}{5}\left(\frac{A\alpha}{2}+\frac{B\beta}{6}+\frac{5A^2}{2B}\right)=\frac{W}{2g}(V)^2 \quad \text{(A-1)}$$

This is accomplished by substituting equations (5) and (6) for the stiffness coefficients, A and B. Simplifying the equation yields:

$$\frac{1}{5}\left(b_0 b_1\alpha+\frac{b_1^2\beta}{3}+5b_0^2\right)=V^2 \quad \text{(A-2)}$$

The equation then is arranged into the quadratic form:

$$Ax^2+Bx+C=0 \quad \text{(A-3)}$$

$$\left(\frac{\beta}{3}\right)b_1^2+(b_0\alpha)\,b_1+5\,(b_0^2-V^2)=0 \quad \text{(A-4)}$$

This allows the use of the quadratic solution:

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A} \quad \text{(A-5)}$$

$$b_1=\frac{-(b_0\alpha)\pm\sqrt{(b_0\alpha)^2-4\left(\frac{\beta}{3}\right)(5)\left(b_0^2-V^2\right)}}{2\left(\frac{\beta}{3}\right)} \quad \text{(A-6)}$$

The negative solution is disregarded as a negative value for the slope, b_1, is a meaningless solution.

$$b_1=\frac{-b_0\alpha+\sqrt{(b_0\alpha)^2-\frac{20\beta}{3}\left(b_0^2-V^2\right)}}{\frac{2}{3}\beta} \quad \text{(A-7)}$$

Conservation of Energy Applied to a F/RMNB Collision

A conservation of energy analysis applied to a F/RMNB collision involves a control volume which begins with the kinetic energy of the vehicle and the barrier at a time immediately prior to the collision. The control volume extends to includes the crush energy and all of the subsequent post collision movements of the vehicle and the barrier to their points of rest.

$$E_{collision}=E_{separation}+E_{crush} \quad \text{(A-8)}$$

Writing the energies in term of speed results in the following:

$$\frac{W_v}{2g}V_{v1}^2+\frac{W_b}{2g}V_{b1}^2=\frac{W_v}{2g}V_{v2}^2+\frac{W_b}{2g}V_{b2}^2+\frac{W_v}{2g}(V_{v,eff})^2 \quad \text{(A-9)}$$

Simplifying and combining like terms yields:

$$W_v(V_{v,eff})^2=W_v(V_{v1}^2-V_{v2}^2)+W_b(V_{b1}^2-V_{b2}^2) \quad \text{(A-10)}$$

Finally solving for the effective speed term yields:

$$V_{v,eff}=\sqrt{\frac{W_b}{W_v}\left(V_{b1}^2-V_{b2}^2\right)+\left(V_{v1}^2-V_{v2}^2\right)} \quad \text{(A-11)}$$