A Method for Determining Accident Specific Crush Stiffness Coefficients

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ABSTRACT

The CRASH3 computer program increasingly is being used by engineers as a tool to reconstruct automobile accidents. The damage analysis portion of CRASH3 provides a useful means for quantifying the change of velocity, $\Delta V$, that was experienced by a vehicle during the collision phase of a traffic accident. The degree of usefulness of the damage analysis portion of the program, however, is dependent upon the availability of valid crush stiffness coefficients.

Published crush stiffness coefficients are available for a large number of vehicles \[1\] & \[2\]. These publications, however, contain only a limited number of coefficients that describe the stiffness characteristics of the side structure of vehicles. Engineers are often asked to perform an accident reconstruction when there are neither published stiffness coefficients for the side structure of an involved vehicle nor crash test data from which to determine the stiffness. Such a collision usually involves the front end structure of a striking "bullet" vehicle and the side structure of a left turning "target" vehicle. If stiffness coefficients are available for the bullet vehicle, then it may be possible to determine accident-specific stiffness coefficients for the target vehicle.

A method is presented in this paper that will allow a determination of accident-specific crush stiffness coefficients for target vehicles. The method is based in theory on the CRASH3 damage algorithm. Intrinsic to the CRASH3 damage algorithm is Newton's Third Law of Motion that states that the forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction and collinear. The collision force exerted on the bullet vehicle is calculated based upon its known stiffness characteristics and the magnitude of its residual crush. The collision force exerted on the target vehicle is set equal to the calculated collision force of the bullet vehicle. Next the damage offset speed, $b_0$, of the target vehicle structure is estimated. An estimation of the damage offset speed is required when mathematically determining the CRASH3 stiffness coefficients, $A$ and $B$ \[3\]. The estimated damage offset speed is then used, with the calculated collision force, to determine the accident-specific stiffness coefficients of the target vehicle. "Hard spots" such as wheel assemblies can be taken into account during this estimation.

The stiffness coefficients determined by this method will be based on the complex circumstances of the specific traffic accident. As a result, the stiffness coefficients determined for target vehicles involved in different accidents may not be based on a common and/or constant set of parameters. These stiffness coefficients, therefore, should be considered valid only for the specific accident.

INTRODUCTION

Engineers are often asked to perform an accident reconstruction when there are neither published stiffness coefficients for the side structure of the target vehicle.
vehicle nor crash test data from which to determine the stiffness. Fortunately, published stiffness coefficients usually are available for the front end structure of the bullet vehicle. This lack of side stiffness data, however, requires an engineer to estimate the stiffness coefficients, $A$ and $B$, of the target vehicle in order to use the damage analysis portion of the CRASH3 program in a reconstruction of such a traffic accident. Such estimations could be associated with a high potential for inaccuracy.

Users of EDCRASH, an upscale PC version of CRASH3, may attempt to use the program’s calculated collision forces as a means to judge the validity of estimated crush stiffness coefficients for the target vehicle. Newton’s Third Law of Motion states that the forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction and collinear. The estimated stiffness coefficients, $A$ and $B$, can be adjusted to produce a balance between the calculated collision forces exerted on each vehicle. The problem with this approach is that there is almost an infinite number of paired $A$ and $B$ coefficients that will result in a balancing of the calculated collision forces. Each of these sets of paired stiffness coefficients will produce different levels of calculated damage energy and ultimately different values for $\Delta V$.

In addition, each of these paired sets of stiffness coefficients will be associated with a different damage offset speed, $b_0$. A damage offset speed, for a given vehicle structure, is the speed at impact with a non-energy absorbing fixed barrier that will produce no residual crush [4]. Potentially, some of these paired sets of coefficients would be associated with damage offset speeds that are so high, or so low, as to be unrealistic. This problem is eliminated by the use of the method presented in this paper. This is because the stiffness coefficients are determined based upon an estimation of the damage offset speed.

The objective of this paper is to provide engineers with an additional tool that can be used to reconstruct traffic accidents. This method for determining accident-specific crush stiffness coefficients, when properly applied, should provide such a tool.

**BACKGROUND**

Vehicular structures are modeled in the CRASH3 damage algorithm as being homogeneous with respect to their stiffness characteristics. Vehicles are divided into three structures, the front, the rear and the side. Each portion of a vehicular structure should have the same stiffness characteristics as any other portion of the same structure. Published crush stiffness coefficients, for example, have been determined based upon this assumption [1] & [2]. This allows the structure to be either treated as one zone, or divided into several zones with the crush stiffness coefficients of each zone being identical.

This model has been generally accepted as being a reasonable simplification with respect to front and rear vehicular structures. The method presented in this paper uses this model of vehicular stiffness for front structures.

Side structures, however, are constructed of many sub-structures that are not nearly homogeneous with respect to their stiffness characteristics. For example, a wheel and suspension assembly has stiffness characteristics that are greatly different from that of a door structure. To account for this difference, side structures are modeled in this method as having homogeneous stiffness within each zone of the damage profile. "Hard spots" such as wheel and suspension assemblies can be isolated from door structures by assigning different zones to each of these structures. This approach is consistent, on a zone by zone basis, with the vehicle model used in the CRASH3 damage algorithm.

This approach also was used in the formulation of the EDCRASH damage algorithm [5]. The EDCRASH damage algorithm takes the CRASH3 damage algorithm one step further and recognizes this variance in stiffness. EDCRASH allows stiffness coefficients to be assigned separately to each zone of the damage profile.

Mathematical determination of the CRASH3 stiffness coefficients, $A$ and $B$, requires an estimation of the damage offset speed for the damaged vehicle structure [3]. Available crash test data can provide some insight and guidance into the selection of a
realistic damage offset speed. Data from 1782 crash tests for vehicle model years 1960-1992 have been analyzed [1]. Low speed impacts of 15 miles per hour, or less, accounted for 416 of these crash tests. From this data, damage offset speeds were determined for the front end structures of 100 vehicles and the rear end structures of 47 vehicles. Statistical analysis of the data revealed a range of 2.0 to 5.0 miles per hour for front end structures. The mean value of the damage offset speeds for the front end structures was 3.9 miles per hour. The damage offset speeds for rear end structures ranged from 3.2 to 4.9 miles per hour. The mean value of the damage offset speeds for the rear end structures was 4.2 miles per hour.

It would be generally expected that side structures of vehicles, such as doors, would tend to have a range of damage offset speeds whose upper end is less than the upper end of the damage offset speeds for front end and rear end structures. Wheel and suspension assemblies would be generally expected to have a range of damage offset speeds whose upper end was greater than the upper end of the damage offset speeds for front end and rear end structures.

PROCEDURE

The proper application of the method requires the presence of the following conditions:

1. The stiffness coefficients for the bullet vehicle are known or can be determined from crash test data.
2. The damage profiles of the bullet vehicle and the target vehicle are known.
3. The damage profiles of both vehicles can be resolved into a direct damage component and, if present, an induced damage component. Direct damage is the portion of the damage profile in which contact occurred between the vehicles. In other words, the vehicle surface where the external collision force was applied. Induced damage is the portion of the damage profile upon which an external collision force was not applied.
4. The direct damages on both vehicles have the same width.
5. The direct damages on the vehicles can be divided into zones such that the zones on the bullet vehicle align with the zones on the target vehicle with which they came into contact. The matching zones are of equal width.

METHOD

The following approach is involved in the application of the method:

1. The direct damages on the vehicles are divided into zones such that the zones on the bullet vehicle align with the zones on the target vehicle with which they came into contact. The matching zones are of equal width.
2. The collision force is determined for each zone of direct damage on the bullet vehicle.
3. The collision force for each zone of direct damage on the target vehicle is set equal to the collision force of the matching zone on the bullet vehicle.
4. A damage offset speed is estimated for each zone of direct damage on the target vehicle.
5. The accident-specific stiffness coefficients are determined for each zone of direct damage on the target vehicle.

Figure 1. Vehicle Configuration at Impact.
The first step entails resolving the damage profiles of the vehicles into their direct damage and induced damage components. Direct damage is the portion of the vehicle surface where the external collision force was applied. Induced damage is the portion of the damage profile upon which an external collision force was not applied.

The direct damage on the target vehicle is divided into zones that segregate the damage based on the stiffness characteristics of the sub-structures. The damage of the bullet vehicle is then divided into zones that align with the zones on the target vehicle with which they came into contact.

The second step involves a determination of the collision force that acted upon each zone of direct damage on the bullet vehicle. This is accomplished by applying the following equation to each zone (see the Appendix for the derivation of the equations):

\[
F_{\text{bullet},j} = \frac{L_{j}}{\cos \alpha_{\text{bullet}}} \left[ A_{\text{bullet},j} + \frac{B_{\text{bullet},j}}{2} \left( C_{\text{bullet},j} + C_{\text{bullet},j+1} \right) \right] \quad \text{[lb]} \quad (1)
\]

Where:
- \( j \) = the zone number,
- \( F_{j} \) = the collision force of the zone, [lb],
- \( L_{j} \) = the width of the zone of direct damage, [in],
- \( \alpha \) = the angle between the Principle Direction of Force (PDOF) and a line normal to the undamaged surface, [degree],
- \( A \) = the stiffness coefficient that represents the maximum force which can be applied per unit width of contact area that will produce no residual crush, [lb/in],
- \( B \) = the stiffness coefficient that represents the ratio of the force per unit width of contact area per unit depth of residual crush, [lb/in^2],
- \( C \) = the crush depth, [in].

In the third step the collision force for each zone of direct damage on the target vehicle is set equal to the collision force of the matching zone on the bullet vehicle.

\[
F_{\text{target},j} = F_{\text{bullet},j} \quad \text{[lb]} \quad (2)
\]

In the fourth step the engineer must estimate a damage offset speed, \( b_{0,\text{target},j} \), for each zone of direct damage on the target vehicle. “Hard spots” such as wheel assemblies can be taken into account during this estimation.

The determined collision forces and the estimated damage offset speeds now can be used to determine the accident-specific value of \( b_{1} \) for each zone of direct damage on the target vehicle.

\[
b_{1,\text{target},j} = b_{0,\text{target},j} + \left[ \frac{2 \cdot F_{\text{target},j} \cdot L_{j} \cdot \cos \alpha_{\text{target},j}}{0.802 \cdot W_{\text{target}} \cdot L_{j}} \frac{C_{\text{target},j} + C_{\text{target},j+1}}{C_{\text{target},j} + C_{\text{target},j+1}} \right] \quad \text{[mph/in]} \quad (3)
\]

Where:
- \( b_{1} \) = the slope of barrier impact speed vs. the residual crush, [mph/in],
- \( b_{0} \) = the damage offset speed, [mph],
- \( L \) = the width of the direct damage, [in],
- \( L_{j} \) = the width of a zone of direct damage, [in],
- \( W \) = the vehicle weight, [lb].

Finally, the accident-specific stiffness coefficients can be determined for each zone of direct damage on the target vehicle.

\[
A_{\text{target},j} = \frac{0.802 \cdot W_{\text{target}} \cdot b_{0,\text{target},j} \cdot b_{1,\text{target},j}^{2}}{L} \quad \text{[lb/in]} \quad (4)
\]

\[
B_{\text{target},j} = \frac{0.802 \cdot W_{\text{target}} \cdot b_{1,\text{target},j}^{2}}{L} \quad \text{[lb/in^2]} \quad (5)
\]

**DISCUSSION**

This paper would not be complete without a discussion of the potential inaccuracies associated with the application of this method. First, the reconstructing engineer should recognize that the potential inaccuracies associated with the determination of the collision forces for the bullet vehicle will be doubled. This occurs when the collision forces for the target vehicle are set equal to the collision forces calculated for the bullet vehicle. Additionally, there exists a potential for inaccuracy in the estimation of a damage offset speed for the zones of direct damage on the target vehicle.

One should recognize that, in reality, the front and rear structures of vehicles are not an ideal homogeneous body. These structures are composed of sub-structures of differing stiffness. Potentially one of
the zones of the damage profile of the bullet vehicle will be stiffer than the structure as a whole. This condition, however, should coexist with a zone that is less stiff than the structure as a whole. The effect of the softer zone on the damage analysis should be counteracted by the effect of the stiffer zone on the analysis. The net effect, therefore, may be minimal.

These potential inaccuracies will propagate and ultimately affect the potential inaccuracies associated with the calculated values of the vehicle damage energies and the $\Delta V$’s experienced by the vehicles. The effect of these potential inaccuracies, however, can be ascertained by performing a sensitivity study. The engineer then can apply an appropriate confidence interval to the reconstruction that will reflect the results of the sensitivity study.

In staged collisions performed at test facilities, a strict protocol is followed and parameters are controlled. Crush stiffness coefficients determined from these tests can be considered to generally represent the structural characteristics of the test vehicle and, therefore, can be used with sound engineering judgment in the reconstruction of traffic accidents. Traffic accidents, on the other hand, are chaotic events involving complex circumstances. The stiffness coefficients determined by this method will be based on the complex circumstances of the specific traffic accident. As a result, the stiffness coefficients determined for target vehicles involved in different accidents may not be based on a common and/or constant set of parameters. These stiffness coefficients, therefore, should be considered valid only for the specific accident.

SUMMARY

1. The method that is presented in this paper for quantifying accident-specific stiffness coefficients for a target vehicle is based on the CRASH3 damage algorithm.
2. Vehicular structures are modeled in the CRASH3 damage algorithm as being homogeneous with respect to their stiffness characteristics. The method presented in this paper uses this model of vehicular stiffness for front structures.
3. This method models side structures as having homogeneous stiffness within each zone of the damage profile. "Hard spots" such as wheel and suspension assemblies are isolated from door structures by assigning different zones to each of these structures. This approach is consistent, on a zone by zone basis, with the vehicle model used in the CRASH3 damage algorithm. This approach also was used in the formulation of the EDCRASH damage algorithm. EDCRASH allows stiffness coefficients to be assigned separately to each zone of the damage profile.
4. The accident-specific stiffness coefficients of the target vehicle are determined based upon the calculated collision force exerted on the bullet vehicle and an estimation of the damage offset speed of the target vehicle. This approach complies with Newton's Third Law of Motion and eliminates the potential for an engineer to inadvertently base a reconstruction on a unrealistic damage offset speed.
5. The stiffness coefficients determined by this method will be based on the complex circumstances of the specific traffic accident. As a result, the stiffness coefficients determined for target vehicles involved in different accidents may not be based on a common and/or constant set of parameters. These stiffness coefficients, therefore, should be considered valid only for the specific accident.
6. The potential inaccuracies associated with this method were discussed. The potential inaccuracies can be ascertained by performing a sensitivity study. An appropriate confidence interval should be applied to the results of the reconstruction based upon the results of the sensitivity study.

REFERENCES

Equation Derivation

The CRASH3 damage algorithm models the structural deformation characteristics of automobiles as a linear spring [6]. That is the force, \( f \), exerted on an automobile per unit width of crush is linearly proportional to the crush.

\[
f = A + B \cdot C \quad \text{[lb/in]} \quad (A-1)
\]

Where:
- \( A \) = the stiffness coefficient that represents the maximum force which can be applied per unit width of contact area that will produce no residual crush, [lb/in],
- \( B \) = the stiffness coefficient that represents the ratio of the force per unit width of contact area per unit depth of residual crush, [lb/in²],
- \( C \) = the crush depth, [in].

From equation (A-1) the collision force for a zone of direct damage can be determined by integrating across the width of the zone using a trapezoidal approximation of the crush profile. This results in equation (A-2).

\[
F_j = L_j \cdot \left[ A_j + \frac{B_j}{2} (C_j + C_{j+1}) \right] \quad \text{[lb]} \quad (A-2)
\]

Where:
- \( j \) = the zone number,
- \( F_j \) = the collision force of the zone, [lb],
- \( L_j \) = the width of the zone, [in].

The Principle Direction of Force (PDOF) often does not act along a line normal to the surface of the vehicle. The crush depth, however, is measured normal to the surface of the vehicle. An adjustment is needed to account for the additional distance over which the surface is displaced while being crushed along this non-normal line of action. The CRASH3 algorithm accounts for this additional distance by multiplying the crush energy by an energy correction factor. This is equivalent to dividing the collision force by the cosine of the angle between the PDOF and a line normal to the involved surface.

\[
F_j = \frac{L_j}{\cos \alpha} \left[ A_j + \frac{B_j}{2} (C_j + C_{j+1}) \right] \quad \text{[lb]} \quad (A-3)
\]

Where:
- \( \alpha \) = the angle between the PDOF and a line normal to the undamaged surface, [degree].

The stiffness coefficients \( A \) and \( B \) in equation (A-3) have been previously defined as follows [3]:

\[
A = \frac{0.802 W b_0 b_1}{L} \quad \text{[lb/in]} \quad (A-4)
\]

\[
B = \frac{0.802 W b_0^2 b_1}{L} \quad \text{[lb/in²]} \quad (A-5)
\]

Where:
- \( W \) = the vehicle weight, [lb],
- \( b_0 \) = the damage offset speed, [mph],
- \( b_1 \) = the slope of barrier impact speed vs. the residual crush, [mph/in],
- \( L \) = the width of the direct damage, [in].

Equations (A-4) and (A-5) apply to the damage profile and the vehicle as a whole. The CRASH3 damage algorithm is based on the assumption that the structure of the vehicle is homogeneous with respect to its stiffness characteristics. Vehicles are divided into 3 structures, the front, the rear and the side. Each portion of a vehicular structure should have the same stiffness coefficients as any other portion of the same structure. This assumption allows equation (A-3) is to be applied independently to each zone of direct damage. In doing so, equations (A-4) and (A-5) need to be rewritten as a function of zone width. This is accomplished by substituting \( L_j \) for \( L \) and by proportioning the vehicle weight to each zone. In order for the sum of the parts to equal the whole, the vehicle weight assigned to each zone must be proportional to the ratio of the width of the zone to the width of the direct damage.

\[
A = \frac{0.802 W \frac{L_j}{L} b_0 b_1}{L_j} = \frac{0.802 W b_0 b_1}{L} \quad \text{[lb/in]} \quad (A-6)
\]
\[ B = \frac{0.802 W \cdot \frac{L_i}{L} \cdot b_{ij}^2}{L} = \frac{0.802 W \cdot b_{ij}^2}{L} \quad \text{[lb/in}^2] \quad (A-7) \]

Substituting equations (A-6) and (A-7) into equation (A-3) and solving for \( b_{ij} \) yields:

\[
b_{ij} = \frac{-b_{ij} + \sqrt{(b_{ij})^2 + \frac{2 \cdot F \cdot W \cdot L \cdot \cos \alpha + C_i + C_{j+1}}}{0.802 \cdot W \cdot L_i \cdot (C_i + C_{j+1})}}{C_i + C_{j+1}} \quad \text{[mph/in]} \quad (A-8)\]
SAMPLE CALCULATIONS

Problem: Determine $\Delta V$ when the stiffness of one of the vehicles is unknown.

Given:
The accident involves a collision where the stiffness is known for the "bullet" vehicle and is unknown for the "target" vehicle.

Constants
\[
g := 32.2 \cdot \frac{\text{ft}}{\text{sec}^2} \quad \frac{(5280 \cdot \text{ft})^2}{(3600 \cdot \text{sec})^2} \cdot \frac{(12 \cdot \text{in})^2}{g \cdot 12 \cdot \text{in} / \text{ft}} = J = 0.802 \cdot \frac{\text{ft}^2 \cdot \text{in}}{\text{mi}^2}
\]

Number of crush measurements, ie $C_1..C_N$
\[
N := 6 \quad i := 0..(N - 1)
\]

Number of crush zones, ie $(N-1)$
\[
j := 0..(N - 2)
\]

V-1: "Bullet" Vehicle Data
\[
C_1 := \begin{array}{c} 7-\text{in} \\ 6-\text{in} \\ 6-\text{in} \\ 10-\text{in} \\ 14-\text{in} \\ 16-\text{in} \end{array} \quad L_1 := \begin{array}{c} 14-\text{in} \\ 14-\text{in} \\ 14-\text{in} \\ 14-\text{in} \\ 14-\text{in} \end{array} \quad wt_1 := 3500-\text{lb}
\]
\[
PDOF_1 := -20\cdot\text{deg} \quad \alpha_1 := -20\cdot\text{deg} \quad OAW_1 := 70\cdot\text{in} \quad X_{F1} := 89.8\cdot\text{in} \quad X_{R1} := 106.4\cdot\text{in}
\]
\[
A_1 := 200\cdot\frac{\text{lb}}{\text{in}} \quad B_1 := 75\cdot\frac{\text{lb}}{\text{in}^2} \quad M_1 := \frac{wt_1}{g} \quad \sum_{j} L_1 = 70\cdot\text{in}
\]

V-2: "Target" Vehicle Data
\[
C_2 := \begin{array}{c} 18-\text{in} \\ 17-\text{in} \\ 15-\text{in} \\ 13-\text{in} \\ 6-\text{in} \\ 5-\text{in} \end{array} \quad L_2 := \begin{array}{c} 14-\text{in} \\ 14-\text{in} \\ 14-\text{in} \\ 14-\text{in} \\ 14-\text{in} \end{array} \quad wt_2 := 3000-\text{lb}
\]
\[
PDOF_2 := 70\cdot\text{deg} \quad \alpha_2 := 20\cdot\text{deg} \quad OAW_2 := 67\cdot\text{in} \quad X_{F2} := 83.3\cdot\text{in} \quad X_{R2} := 91.6\cdot\text{in}
\]
\[
M_2 := \frac{wt_2}{g} \quad \sum_{j} L_2 = 70\cdot\text{in}
\]
Calculations:

Determine the coefficients \( b_0 \) and \( b_1 \) for the "bullet" vehicle.

\[
b_{1\_j} = \sqrt{\frac{B_{1\_j} \cdot L_{1\_total}}{J\cdot wt_{1\_j}}} \]

\[
b_{0\_j} = \sqrt{\frac{A_{1\_j} \cdot L_{1\_total}}{J\cdot wt_{1\_j} \cdot b_{1\_1}}}
\]

\[
b_{1\_1} = \begin{bmatrix} 1.37 \\ 1.37 \\ 1.37 \\ 1.37 \\ 1.37 \end{bmatrix} \text{ mph} \]

\[
b_{0\_1} = \begin{bmatrix} 3.6 \\ 3.6 \\ 3.6 \end{bmatrix} \text{ mph}
\]

Calculate the collision force exerted on the "bullet" vehicle.

\[
F_{1\_j} = \frac{L_{1\_j}}{\cos(\alpha_{1\_j})} \left[ A_{1\_j} + \frac{B_{1\_j}}{2} \left( C_{j} + C_{1\_j+1} \right) \right] \]

\[
F_{1\_j} = \begin{bmatrix} 10243 \\ 9684 \\ 11919 \\ 16388 \\ 19740 \end{bmatrix} \text{ lb}
\]

\[
F_{1\_total} = \sum_{j} F_{1\_j}
\]

Estimate the damage offset speed for each crush zone on the "target" vehicle.

\[
b_{0\_2\_j} := \begin{bmatrix} 2 \text{ mph} \\ 2 \text{ mph} \\ 2 \text{ mph} \\ 2 \text{ mph} \\ 8 \text{ mph} \end{bmatrix}
\]

NOTE: Zone 1 through 4 is a door structure.

Zone 5 is a wheel and suspension assembly.

These values are part of a fictitious example problem and do not necessarily represent the author's opinion regarding realistic damage offset speeds.

Calculate the accident specific stiffness coefficients for the "target" vehicle.

Set:

\[
F_{2\_j} := F_{1\_j}
\]

\[
b_{0\_2\_j} := \begin{bmatrix} \left( b_{0\_2\_j} \right)^2 + \frac{2 \cdot F_{2\_j} \cdot L_{1\_total} \cdot \cos(\alpha_{2\_j})}{J\cdot wt_{2\_j} \cdot L_{1\_j}} \cdot \left[ C_{2\_j} + C_{2\_j+1} \right] \end{bmatrix}
\]

\[
b_{1\_2\_j} := \begin{bmatrix} 1.22 \\ 1.73 \\ 2.02 \end{bmatrix} \text{ mph} \]

\[
A_{2\_j} := \frac{J\cdot wt_{2\_j} \cdot b_{0\_2\_j} \cdot b_{1\_2\_j}}{L_{1\_total}}
\]

\[
A_{2} = \begin{bmatrix} 69.7 \\ 70.5 \\ 83.8 \\ 119.1 \\ 554.9 \end{bmatrix} \text{ lb} \]
Verify the results by calculating the collision force for the "target" vehicle.

\[
F_2 := \frac{L_2}{\cos(\alpha_2)} \left[ A_2 + \frac{B_2}{2} \left[ C_2 + C_{2(i+1)} \right] \right]
\]

\[
F_2_{\text{total}} := \sum_j F_2_j
\]

Calculate the damage energy for both vehicles.

\[
E_1 := \frac{L_1}{\cos(\alpha_1)} \left[ \frac{A_1}{2} \left( C_1 + C_{1(i+1)} \right) + \frac{B_1}{6} \left[ (C_1)^2 + C_1 \cdot C_{1(i+1)} + (C_{1(i+1)})^2 \right] \right] + \frac{(A_1)^2}{2 \cdot B_1}
\]

\[
E_1_{\text{total}} := \sum_j E_1_j
\]

\[
E_1 = \begin{bmatrix} 4167.3 \\ 3721.4 \\ 5703.3 \\ 10723.9 \\ 15480.3 \end{bmatrix} \cdot \text{ft-lb}
\]

\[
E_1_{\text{total}} = 39796 \cdot \text{ft-lb}
\]

\[
E_2 := \frac{L_2}{\cos(\alpha_2)} \left[ \frac{A_2}{2} \left( C_2 + C_{2(i+1)} \right) + \frac{B_2}{6} \left[ (C_2)^2 + C_2 \cdot C_{2(i+1)} + (C_{2(i+1)})^2 \right] \right] + \frac{(A_2)^2}{2 \cdot B_2}
\]

\[
E_2_{\text{total}} := \sum_j E_2_j
\]

\[
E_2 = \begin{bmatrix} 8846 \\ 7714.8 \\ 8276.3 \\ 8020.2 \\ 8290.5 \end{bmatrix} \cdot \text{ft-lb}
\]

\[
E_2_{\text{total}} = 41148 \cdot \text{ft-lb}
\]
Verify the method by the reversing of the process.

Use the accident specific stiffness coefficients for the "target" vehicle as the known stiffness and calculate the stiffness coefficients, the collision force and the damage energy for the "bullet" vehicle. This should result in the same values that were used/calculated for the "bullet" vehicle at the beginning of this process.

\[
b_{1-1} = \frac{-b_{0-1} + \sqrt{(b_{0-1})^2 + \frac{2 \cdot F_2 \cdot L_{1\text{total}} \cdot \cos(\alpha_1)}{J \cdot \text{wt} \cdot L_1 \cdot (C_1 + C_{1(i+1)})}}}{C_1 + C_{1(i+1)}} \]

\[
b_{1-1} = \begin{bmatrix} 1.37 \\ 1.37 \\ 1.37 \\ 1.37 \\ 1.37 \end{bmatrix} \text{ mph/in}
\]

\[
A_j = \frac{J \cdot \text{wt} \cdot b_{0-1} \cdot b_{1-1}}{L_{1\text{total}}}
\]

\[
B_j = \frac{J \cdot \text{wt} \cdot (b_{1-1})^2}{L_{1\text{total}}}
\]

\[
F_j = \frac{L_1}{\cos(\alpha_1)} \left[ A_1 + \frac{B_j}{2} \left[ C_1 + C_{1(i+1)} \right] \right]
\]

\[
F_{1\text{total}} := \sum_j F_j
\]

\[
E_j = \frac{L_1}{\cos(\alpha_1)^2 \cdot \left( \frac{12 \cdot \text{in}}{\text{ft}} \right)^2} \left[ \frac{A_j}{2} \left( C_1 + C_{1(i+1)} \right) + \frac{B_j}{6} \left[ (C_1)^2 + C_1 \cdot C_{1(i+1)} + (C_{1(i+1)})^2 \right] + \frac{(A_j)^2}{2 \cdot B_j} \right]
\]

\[
E_j = \begin{bmatrix} 4167.3 \\ 3721.4 \\ 5703.3 \\ 10723.9 \\ 15480.3 \end{bmatrix} \text{ ft-lb}
\]

\[
E_{1\text{total}} := \sum_j E_j
\]

\[
E_{1\text{total}} = 39796 \text{ ft-lb}
\]