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Speed from Skids: A Modern Approach

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ABSTRACT

An automobile equipped with a conventional brake system often will produce four skid marks on a roadway surface during maximum braking. This condition often occurs immediately prior to a collision in a traffic accident. Knowing the length of the skid marks, S_S, and using the dynamic coefficient of friction for the roadway surface, μ, a reconstructing engineer can determine the amount of kinetic energy converted to work during the skidding process on a level roadway. The equation used in this process states that, the portion of the kinetic energy of the vehicle that was used to perform the work of slowing the vehicle is equal to the braking force applied to the vehicle through the skidding distance. Solving the equation for the speed of a vehicle that skids to a stop yields, $V_S = \sqrt{2 \cdot g \cdot \mu \cdot S_S}$, the traditional speed from skids equation.

Problems exist with the traditional speed equation that limit its practical use in traffic accident reconstruction. A major problem is that the equation does not account for the energy converted to work during the transient portion of the braking process. The transient portion of braking occurs prior to the onset of visible skid marks on the roadway surface.

A new speed from skids equation is set forth in this paper that will allow the reconstructing engineer to determine the speed of a vehicle at the onset of maximum braking.

In addition, new equations are presented that determine the total braking distance and total braking time. An empirical investigation was used to gather data on a test vehicle. This data was used in a comparison of the accuracy of the new method and the traditional method in estimating the speed, braking distance and braking time of the test vehicle.

INTRODUCTION

The traditional equation used to determine the speed of a vehicle that skids to a stop states that, the kinetic energy of the vehicle at the onset of skidding is equal to the work performed by the braking force applied to the vehicle through the skidding distance.

$$V_S = \sqrt{2 \cdot a \cdot S_S} \qquad (1)$$

Where:

V_S = the speed of the vehicle at the onset of skidding,
a = the deceleration experienced by a vehicle during the skidding process,
S_S = the distance traveled by the vehicle while producing visible skid marks on the roadway surface.

The deceleration experienced by a vehicle during a braking process is the product of a drag factor, f, and the acceleration of gravity, g.

$$a = f \cdot g \qquad (2)$$

The drag factor is a function of the dynamic coefficient of friction of the roadway surface, μ, and the grade of the roadway, G. For small roadway grades, the dynamic coefficient of friction can be determined by the following equation.

$$\mu = f \pm G \qquad (3)$$

If the vehicle was traveling uphill during the skidding process, then the dynamic coefficient of friction can be determined by subtracting the grade from the drag factor. If the vehicle was traveling downhill during the skidding process, then the grade of the roadway is added to the drag factor. If the roadway was level, then the dynamic coefficient of friction is equal to the drag factor.

Equation (1) is based upon the First Law of Thermodynamics which states that energy can not be created or destroyed. The First Law states that the energy of a "system" is equal to the energy entering the "system" plus the work performed on the surroundings minus the energy leaving the "system." A "system" is defined by a control volume which is established as a physical region on the roadway.

In the traditional speed equation, entering energy is the kinetic energy of the vehicle at the onset of skidding. If the vehicle brakes to a stop within the control volume, then all of the kinetic energy has been used to perform work. If the vehicle exits the control volume, then the work performed is the difference between the entering kinetic energy and the exiting kinetic energy. Provided there is no collision within the control volume, the work performed is solely through the braking action.

Problems exist with the traditional speed equation that limit its practical application in traffic accident reconstruction. The equation is based upon an instantaneous lockup of all four brakes and the instantaneous production of skid marks*[1] [2] [3]. In other words, the speed of a vehicle at the onset of braking is equal to the speed of the vehicle at the point in time that visible skid marks were first deposited on the roadway surface. The brakes, however, do not lockup at the instant a force is applied to the brake pedal [1] [2] [3] [4] [5] [6]. Instead, a transient braking process occurs. The transient process begins with the application of the brake pedal and ends at the onset of impending skid marks on the roadway surface. The initial speed of a vehicle at the onset of braking, therefore, is faster than the speed of that vehicle at the onset of skidding.

Equation (1) does not account for the energy that is used to perform work during the transient portion of the braking process. The kinetic energy used to perform the work of slowing a vehicle during the transient portion of maximum braking has been estimated previously to be 15% to 25% of the initial kinetic energy of the vehicle [4] [5].

It has been suggested that adding this transient braking energy to the kinetic energy at the onset of skidding could improve the accuracy of a speed determination [5]. This approach assumes the transient energy is a fixed percentage of the initial kinetic energy of the skidded vehicle. The speed reduction which occurs during a given transient braking process, however, is essentially a constant amount and is independent of the initial speed. The transient braking energy, therefore, will be a greater percentage of the initial kinetic energy at slower initial speeds than at faster initial speeds. As a result, this approach may not accurately determine the speed at the onset of braking.

A second problem becomes apparent when attempting to determine the dynamic coefficient of friction of the roadway surface. In the traditional method, the deceleration and the dynamic coefficient of friction are 'derived' values [1] [2] [3]. A skid test is performed where the brakes of an exemplar vehicle, that is traveling at a known speed, are locked. The vehicle is allowed to skid to a stop and the lengths of the visible skid marks are measured. If the roadway is not level, then the grade of the roadway is measured. The speed of the vehicle, the skidding distance and the grade of the roadway are the 'measured' values that are used to 'derive' the coefficient of friction. Equations (1), (2) and (3) are then rearranged, solving for the dynamic coefficient of friction as a function of these 'measured' values.

$$\mu = \frac{V_S^2}{2 \cdot g \cdot S_S} \mp G \qquad (4)$$

On the surface, the traditional approach used to determine the dynamic coefficient of friction of a roadway surface seems to be relatively simple and straight forward. Accounting for the transient braking process, however, creates a problem in the proper application of the First Law of Thermodynamics. The traditional method matches the speed of the exemplar vehicle at the onset of braking with the skidding distance. This is an improper application of the First Law of Thermodynamics. The result is a false indication that the dynamic coefficient of friction changes with vehicle

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[*] The numbers in the brackets refer to references listed at the end of the paper.

speed. (See the appendix for further discussion of this problem).

A more modern method for determining the dynamic coefficient of friction is to perform a skid test with an exemplar vehicle using an on-board accelerometer. If an appropriate accelerometer has been selected, then this process will accurately measure the deceleration of the vehicle. The deceleration is a 'measured' value in this method, rather than a 'derived' value as in the traditional method. To properly apply this data to a reconstruction using the traditional speed equation, the quasi-steady state deceleration which was experienced only during the skidding process is averaged. The transient portion of the deceleration curve should not be included in the averaging process. This average deceleration is then applied in the traditional speed equation using the skid length. Use of this approach will solve the second problem associated with the traditional speed equation. It does not, however, account for the transient braking process and, therefore, will not determine the speed of the subject vehicle at the onset of braking. This approach also will not accurately determine the total braking distance nor total braking time.

A new speed from skids equation, that will allow the reconstructing engineer to determine the speed of a vehicle at the onset of maximum braking, is set forth in this paper. In addition, new equations for braking distance and braking time are presented. This new method takes into account the transient braking period and, therefore, solves the first problem associated with the traditional equation. When used in conjunction with accelerometer data, the new equations also solve the second problem associated with the traditional method. As a result, the new equations more accurately estimate speed, braking distance and braking time.

An empirical investigation was used to gather data on a test vehicle. This data was used to compare the accuracy of the new method and the traditional method in estimating the speed, braking distance and braking time of the test vehicle.

NEW METHOD

EQUATION DERIVATION

A characteristic deceleration curve of locked wheel braking as a function of time is illustrated in Figure A1. The curve can be separated into two regions. In the first region, the brakes are applied and the deceleration experienced by the vehicle increases to a maximum rate. The vehicle then transitions into a quasi-steady state condition where the brakes are locked. This quasi-steady state deceleration continues until the end of the locked wheel braking.

In the new method, the transient period of braking is modeled by a constant rate of change of deceleration [1]. This modeling results in a triangular area bounded by the onset of braking, t_0, and the point in time, t_S, where the vehicle begins to leave visible impending skid marks on the surface of the roadway. This model is shown in Figure A2.

$$a(t) = \frac{a}{t_S} \cdot t \qquad \text{for: } t_0 \le t \le t_S \text{, with } t_0 = 0 \qquad (5)$$

Where:

$a =$ the average deceleration during the quasi-steady state portion of braking.

In the second region, the quasi-steady state portion of the curve is modeled as a constant rate of deceleration. The quasi-steady state deceleration forms a rectangular area bounded by the time of the onset of tire marks, t_S, and the point in time, t_T, at which the vehicle comes to rest.

$$a(t) = a \qquad \text{for: } t_S \le t \le t_T \qquad (6)$$

The height of the triangular, and rectangular, areas are set equal to the average value of deceleration from time t_S to t_T. On a level roadway, the average deceleration is equal to the product of the dynamic coefficient of friction and the acceleration of gravity.

Integrating equations (5) results in an equation for vehicle speed during the transient period. Adding this equation for the transient period to equation (1), which represents the quasi-steady state period, results in the new equation for speed from skids. If the vehicle was involved in a collision before it came to a stop, then the control volume used in the conservation of energy ends at the moment of contact. The energy departing the control volume is the kinetic energy of the vehicle at impact. This departing energy is expressed in the equation as the square of the impacting speed, V_C.

$$V = \sqrt{V_C{}^2 + 2 \cdot a \cdot S_S} + \frac{1}{2} \cdot a \cdot t_S \qquad (7)$$

Integrating again results in an equation for the total braking distance.

$$S_T = S_S + \sqrt{V_C^2 + 2 \cdot a \cdot S_S} \cdot t_S + \frac{1}{3} \cdot a \cdot t_S^2 \qquad (8)$$

The total braking time equation is derived from the kinematics of rectilinear motion.

$$t_T = \frac{V_S - V_C}{a} + t_S \qquad (9)$$

EMPIRICAL INVESTIGATION

A 1989 Toyota Camry sedan equipped with front disc brakes and rear drum brakes was used as the test vehicle. The brake system of the test vehicle was inspected and adjusted for optimal performance. Tire inflation was maintained according to the tire manufacturers specification. An on-board vehicle computer data acquisition system was used to record data from six different vehicle sensors. A non-contact optical sensor was used to measure vehicle speed and the distance traveled during the braking process. Four wheel pulse transducers were used to measure the speed and the distance traveled by each wheel. A uni-axial accelerometer was used to measure vehicle deceleration. The accelerometer had a range of +/- 2.0 g, a frequency response of 100 hertz and was filtered at 5 hertz. A brake switch was attached to the brake pedal that initiated a data set at the instant a force was applied to the brake pedal. (See the appendix for a list of the test equipment).

Figure 1. Test Vehicle

A total of thirty-two brake runs were successfully completed at speeds of 20, 30, 40, 50, and 60 miles per hour on straight and level roadways. Roadway surfaces selected for testing were limited to new asphalt, traveled asphalt, and new chip-sealed surfaces.

Results

Upon the application of maximum pedal pressure associated with an emergency braking maneuver, the test vehicle experienced an amount of transient braking prior to producing impending skid marks. The transient brake time ended at the point in time where the first impending skid mark could be seen on the surface of the roadway. The results of the brake runs show that each wheel of the test vehicle was capable of behaving independently of the other three wheels. Wheel lock-up was observed to be neither instantaneous nor simultaneous for all four wheels. As a result, a minimum transient brake time was determined for each test configuration.

The roadways used for the testing were level. The dynamic coefficient of friction, therefore, was determined by dividing the deceleration curve by the acceleration of gravity and averaging the resultant values of the quasi-steady state portion of the curve. The dynamic coefficient of friction, as illustrated in the deceleration curves of Figure A3, appeared to generally have a slight downward trend with increasing speed. The trend, however, is not statistically significant due to the small sample population size and the variance of the data. For the practical purposes of accident reconstruction, however, the coefficient of friction is independent of vehicle speed. A graph of the 'measured' dynamic coefficient of friction versus vehicle speed is shown Figure A4.

Figure A5 shows that the minimum transient brake times also were found to be independent of the speed of the vehicle. The dynamic coefficient of friction did not have an affect on the transient brake times. This is shown in Figure A6. This indicates that the new equations can be used in a reconstruction without regard to the speed of the vehicle or the dynamic coefficient of friction of the roadway surface.

The minimum transient brake times, shown in Figure A5, were found to be a function of the type of roadway surface. The condition of the roadway surface affected the ability to see the onset of the skid marks. New asphalt, which was clean, uniform in coloration and devoid of stains or discoloration provided the best surface on which to locate the onset of impending skid marks. Locating impending skid marks on traveled asphalt was more difficult. Locating the impending skid marks on new chip-sealed surfaces was the most difficult. The minimum transient brake time on new asphalt was approximately 0.11 seconds. The test vehicle had a minimum transient brake time of approximately 0.18 seconds on a traveled asphalt surface. On a new chip-sealed surface, the test vehicle had the largest transient brake time of approximately 0.25 seconds.

The minimum transient braking times are fixed/constant values for each roadway surface type. When a minimum transient braking time is used in the new equations, the velocity change that is determined to have occurred during the transient braking process also will be a fixed/constant amount for the given type of roadway surface. This can be seen in Figure A2 where the velocity change during the transient braking process is equal to the area of the triangle bounded by the onset of braking, t_0, and the point in time, t_S, where the transient braking process ends.

As indicated earlier, the brake system of the test vehicle was inspected and adjusted for optimal performance. Any under adjustment of the brakes of a vehicle would tend to lengthen the transient braking process. The use of a minimum transient brake time, in a reconstruction where the subject vehicle’s brake were not performing at optimum, will result in an under estimation of the speed of the subject vehicle. The magnitude of the under estimation will be proportional to the difference in the actual transient braking time and the minimum transient braking time.

A test vehicle was configured with a G-Analyst accelerometer during a portion of the test runs. The G-Analyst measurements of deceleration were found to be over-damped as shown in Figure A7. The G-Analyst, however, was found to accurately measure the deceleration during the quasi-steady state portion of the braking process when the test speed was 40 miles per hour or greater. At test speeds less than 40 miles per hour, the over-damped G-analyst was not able to consistently respond quickly enough to accurately measure the relatively short-lived quasi-steady state deceleration.

DISCUSSION

METHOD COMPARISON

Figures A8, A9, and A10 display the results of a comparison of the new method and the traditional method. Each method was used to estimate the speed, braking distance and braking time of the test vehicle. The results were then compared to the measured test data. From the figures, we can see that the new method consistently calculates speed, distance, and time values with a smaller percentage difference from the measured test values than the traditional method. The new method more accurately determines the speed of the vehicle, the braking distance and the braking time than the traditional approach.

An example reconstruction and avoidability study was performed to illustrate the difference between the traditional method and the new method (see appendix). A hypothetical accident involving a pedestrian and a 1989 Toyota Camry was reconstructed. The pedestrian enters the roadway, sees the oncoming car and becomes paralyzed with fear. The driver of the car sees the stopped pedestrian and applies the vehicle’s brakes in an emergency braking maneuver. The car, however, collides with the pedestrian while traveling at a speed of 37 miles per hour. Prior to the collision, the car left 59 feet of skid marks on the dry surface of a traveled asphalt roadway. The dynamic coefficient of friction was 0.78 and the transient braking time was 0.18 seconds. The driver of the car is assumed to have experienced a perception-decision-reaction (PDR) process of 1.5 seconds in duration. The speed limit for the roadway is 45 miles per hour.

Using the traditional approach, the speed of the car at the onset of the skid marks was determined to have been approximately 52 miles per hour. The new method resulted in a speed at the onset of braking of 54 miles per hour. The car was determined, by the traditional method, to have been located approximately 173 feet from the point of impact (POI) at the onset of the PDR process of the driver. The new method placed the car approximately 192 feet from the POI at the onset of the PDR process. Now the question has been asked, “If the car had been traveling at the speed limit, would the collision still have occurred?”. The traditional approach indicated that the car would have skidded to a stop approximately 13 feet beyond the POI and the collision still would have occurred. The new method more accurately indicated that the car would have braked to a stop at the POI and the collision would not have occurred.

It is apparent that the new method can have a dramatic affect on the results of a reconstruction. The usefulness of this new method, however, is limited by the availability of measured minimum transient brake times. In order to expand the application of the new equations, further work is needed to determine the minimum transient brake times for other vehicles. It is possible that additional work could determine that a relationship exists between various types of brake systems and the minimum transient brake time. Should such a relationship exist, it may be possible to form a generalization that will allow for the establishment of a standardized minimum brake time that can be applied to untested vehicles.

NEW METHOD APPLICATION

When the minimum transient braking time for a given vehicle has been determined, then the equations (7), (8) and (9) can be used to determine the velocity at the onset of braking, the total braking distance and total braking time more accurately that by the traditional method. The deceleration experienced by a subject vehicle in a reconstruction can be either determined by estimation or by measurement. If measured, an appropriate accelerometer should be used to determine the deceleration curve.

It has been suggested previously that double integration of a deceleration curve can be used to determine the distance traveled by a test vehicle during a maximum braking application [8]. Such a process is not recommended due to the propagation of inaccuracy associated with the instrumentation. The location of t_S on the deceleration curve can be accurately determined only with the use of additional distance measuring equipment. If this equipment is not available, then the location of t_S can be estimated for the purposes of determining the average deceleration of the quasi-steady state. Our testing of the 1989 Toyota Camry suggests that the location of t_S can be estimated at the peak of the initial rise in the deceleration curve.

These equations also should work well when applied to vehicles equipped with air brake systems that have a pneumatic lag time. The longer the duration of the transient brake time, the greater the accuracy difference will be between the new method and the traditional approach.

The authors also have observed that, some vehicles that are equipped with anti-lock brake systems (ABS) will leave skid marks on the surface of a roadway during maximum braking. Some of these vehicles leave faint impending skid marks while others leave skid marks nearly as dark as those from non-ABS equipped vehicles. The new equations should allow these skid marks to be analyzed more accurately than with the traditional approach, once these vehicles have been tested to determine their transient braking times.

SUMMARY

1. A new speed from skids equation is set forth in this paper. The equation allows the reconstructing engineer to determine the speed of a vehicle at the onset of maximum braking. In addition, new equations for braking distance and braking time were presented.
2. Minimum transient brake times were determined for a 1989 Toyota Camry sedan. The minimum transient brake time was found to be a function of the roadway surface. The minimum transient brake time was found to be approximately 0.11 seconds for new asphalt, 0.18 seconds for traveled asphalt and 0.25 seconds for a new chip-sealed surface.
3. The brake system of the test vehicle was inspected and adjusted for optimal performance. Any under adjustment of the brakes would tend to lengthen the transient braking process.
4. On the surface, the traditional approach used to determine the dynamic coefficient of friction of a roadway surface seems to be relatively simple and straight forward. Improper application of the First Law by the traditional method, however, results in a false indication that the dynamic coefficient of friction changes with vehicle speed.
5. The dynamic coefficient of friction was found to be essentially independent of vehicle speed. The minimum transient brake times also were found to be independent of vehicle speed and the coefficient of friction. This indicates that the new equations can be used in a reconstruction without regard to the speed of the vehicle or the dynamic coefficient of friction of the roadway surface.

6. The G-Analyst measurements of acceleration were found to be over-damped. The G-Analyst was found to accurately measure the deceleration during the quasi-steady state portion of the braking process when the test speed was 40 miles per hour or greater.
7. The usefulness of the new equations as a reconstructing tool was demonstrated. Additional work is required to determine the minimum transient brake times for other vehicles.
8. Additional analysis could result in the ability to make a generalization regarding minimum transient brake times for various types of brake systems (i.e. front disc/rear disc, front disc/rear drum, front drum/rear drum, etc.).
9. This new method should work well when applied to a reconstruction involving vehicles equipped with air brakes. The longer the duration of the transient brake time, the greater the accuracy difference will be between the new method and the traditional approach.
10. Observations have been made indicating some vehicles equipped with ABS leave skid marks on the surface of the roadway during maximum braking. When transient brake times are determined for these vehicles, the new equations should allow for more accurate analysis of the skid marks than the traditional method.

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APPENDIX

FURTHER DISCUSSION

In the traditional method, the deceleration and the dynamic coefficient of friction are 'derived' quantities. In other words, the dynamic coefficient of friction is determined by measuring other parameters which are used mathematically to solve for the coefficient. The speed of the vehicle at the onset of braking, the skidding distance and the grade of the roadway are the 'measured' values that are used to 'derive' the coefficient of friction.

When the skidding distance is used in equation (4), the beginning of the control volume is set at the onset of the visible skid marks. As a result, the exemplar vehicle enters the control volume not at the onset of braking speed, rather it enters at a somewhat reduced speed due to the occurrence of the transient braking process. To properly apply the First Law of Thermodynamics, the speed of the exemplar vehicle at the onset of the visible skid marks should be used with the skidding distance in equation (4). Without test equipment that continuously measures the vehicle speed, however, the only speed known is the speed at the onset of braking.

The traditional method improperly matches the speed of the exemplar vehicle at the onset of braking with the skidding distance. As a result, the traditional method will not determine the dynamic coefficient of friction. Rather it will determine a value for an 'experimental constant'.

$$C = \frac{V^2}{2 \cdot g \cdot S_S} \mp G \qquad (10)$$

Where:

V = the speed of the vehicle at the onset of braking,

S_S = the distance traveled by the vehicle while producing visible skid marks on the roadway surface.

The 'experimental constant' will tend to be greater in magnitude than the actual dynamic coefficient of friction.

$$C > \mu \qquad (11)$$

This is due to the speed at the onset of braking being greater than the speed at the onset of skidding. Since the speed change experienced during a given transient braking process is essentially a constant amount, the transient braking energy will be a greater percentage of the initial kinetic energy of the exemplar vehicle at slower initial speeds than at faster initial speeds. This relationship is shown in Figure A11. The magnitude of the 'experimental constant', therefore, will be greater at slower initial speeds and lessor at faster initial speeds.

$$C_{20} >> C_{60} \qquad (12)$$

This creates a false indication that the dynamic coefficient of friction is dependent on the speed of the vehicle.

The data gathered during the skid testing of the 1989 Toyota Camry was used to illustrate the false indication of a non-constant dynamic coefficient of friction created by the use of the traditional method. The difference between the 'measured' dynamic coefficient of friction and the 'derived' 'experimental constant' are shown in Figure A12. At an onset of braking speed of 20 miles per hour, the average 'experimental constant' is 1.25 and the average dynamic coefficient of friction is 0.82 for traveled asphalt. The 'experimental constant' is 52% greater than the 'measured' dynamic coefficient. As the onset of braking speed of 60 miles per hour, the 'experimental constant' is only 8% greater than the dynamic coefficient of friction.

The deceleration measured during the quasi-steady state portion of the braking process also can be used to illustrate that the indication of a speed dependent coefficient of friction is false. If the dynamic coefficient of friction were in fact 1.25 at 20 miles per hour as shown in Figure A11, then the deceleration curves in Figure A3 should indicate a steady state deceleration of approximately 1.25 times gravity at a skidding speed of 20 miles per hour. The curves, however, clearly show a steady state deceleration of approximately 0.75 times gravity at this speed. The deceleration curves also show that the deceleration is essentially constant during the quasi-steady state portion of braking. The dynamic coefficient of friction, clearly, is not dependent on the speed of the vehicle.

EQUIPMENT LIST

1. The test vehicle:
   a. A 1989 Toyota Camry DX four door sedan equipped with front wheel drive, automatic transmission, front disc brakes, rear drum brakes, and 185/70SR14 Steel Belted Radial tires.
2. Vehicle test equipment:
   a. A Datron 'EEP-2' on-board computer data acquisition system.
   b. A Datron 'L3' non-contact optical sensor for speed and distance measurement.
   c. Four Datron 'WPT-1000' wheel pulse transducers for wheel speed and distance measurement.
   d. A Schaevitz '433' accelerometer for vehicle deceleration measurement.
   e. A brake pedal force switch for determining onset of brake pedal application.
   f. Valentine Research 'G-Analyst' accelerometer for vehicle deceleration measurement.

FIGURES

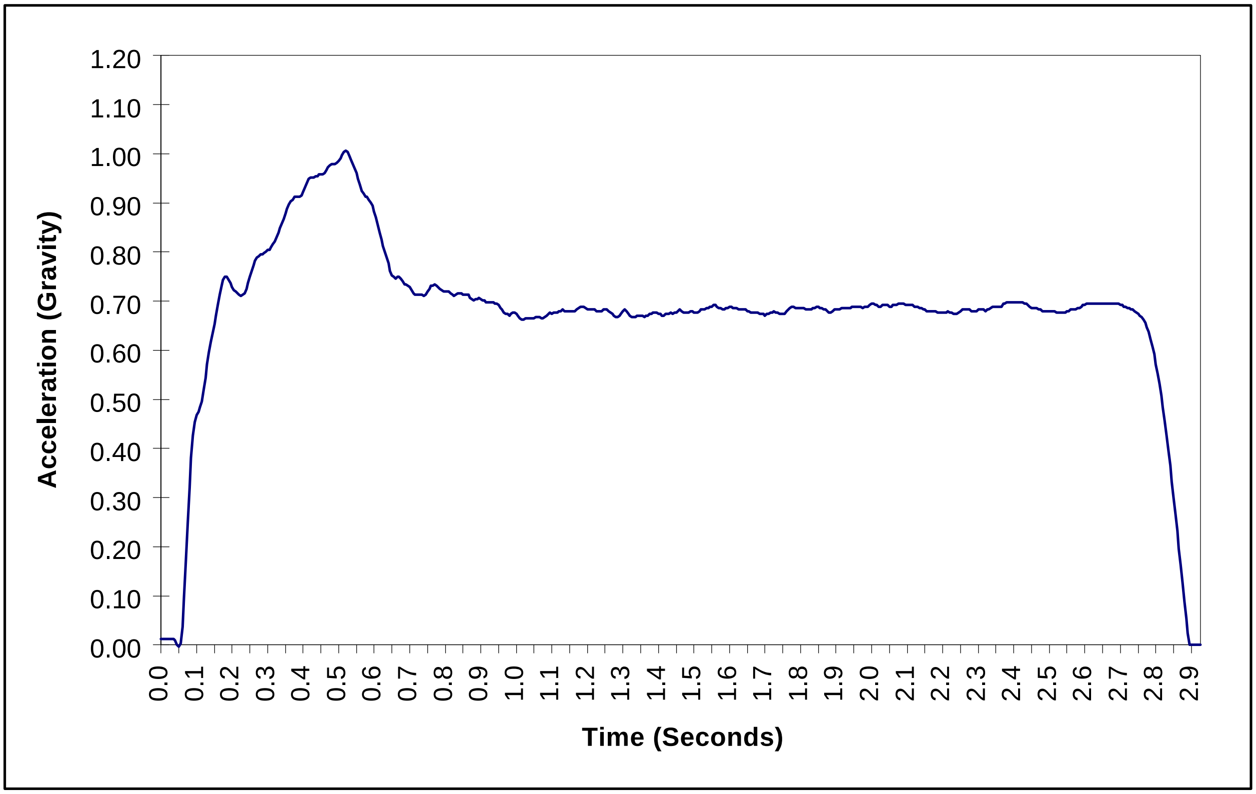

Figure A1. Characteristic Deceleration Curve for Locked Wheel Braking.

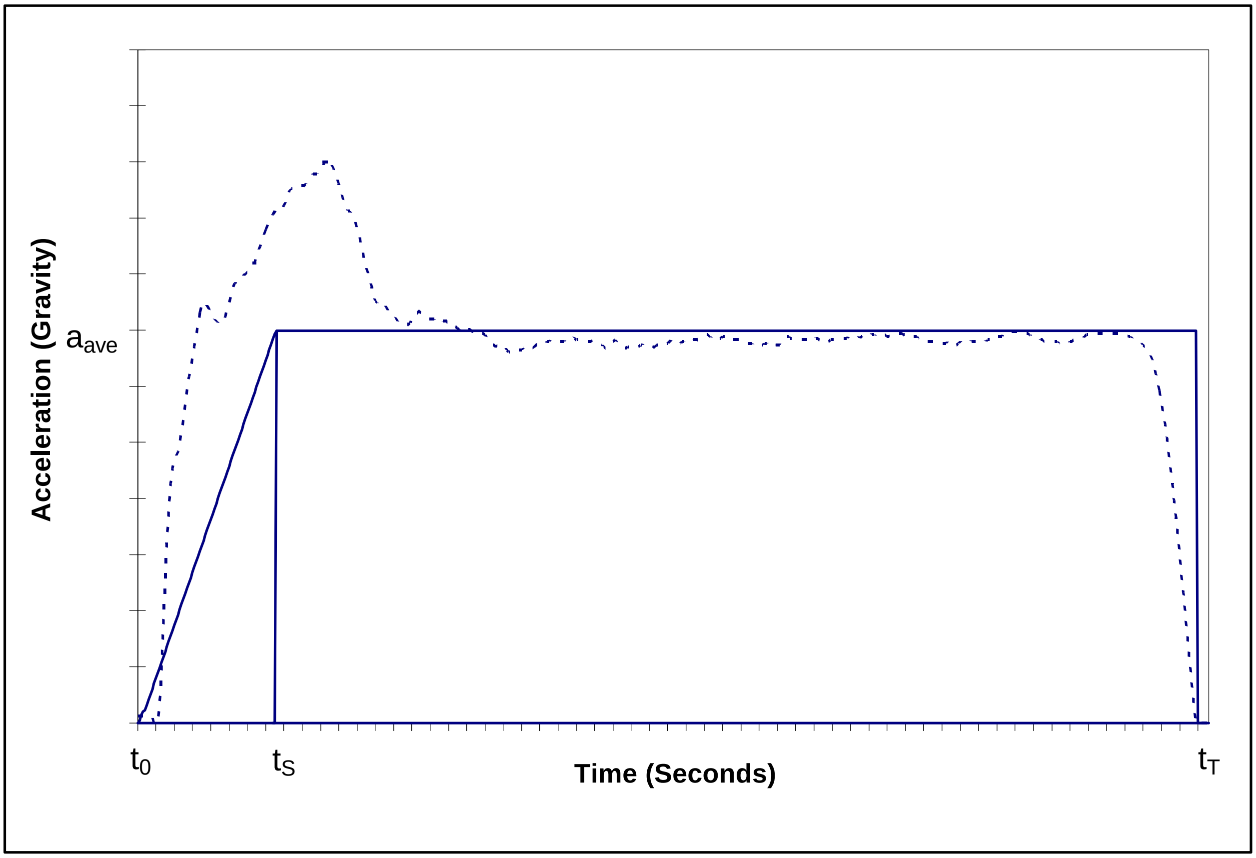

Figure A2. The New Model for Locked Wheel Braking.

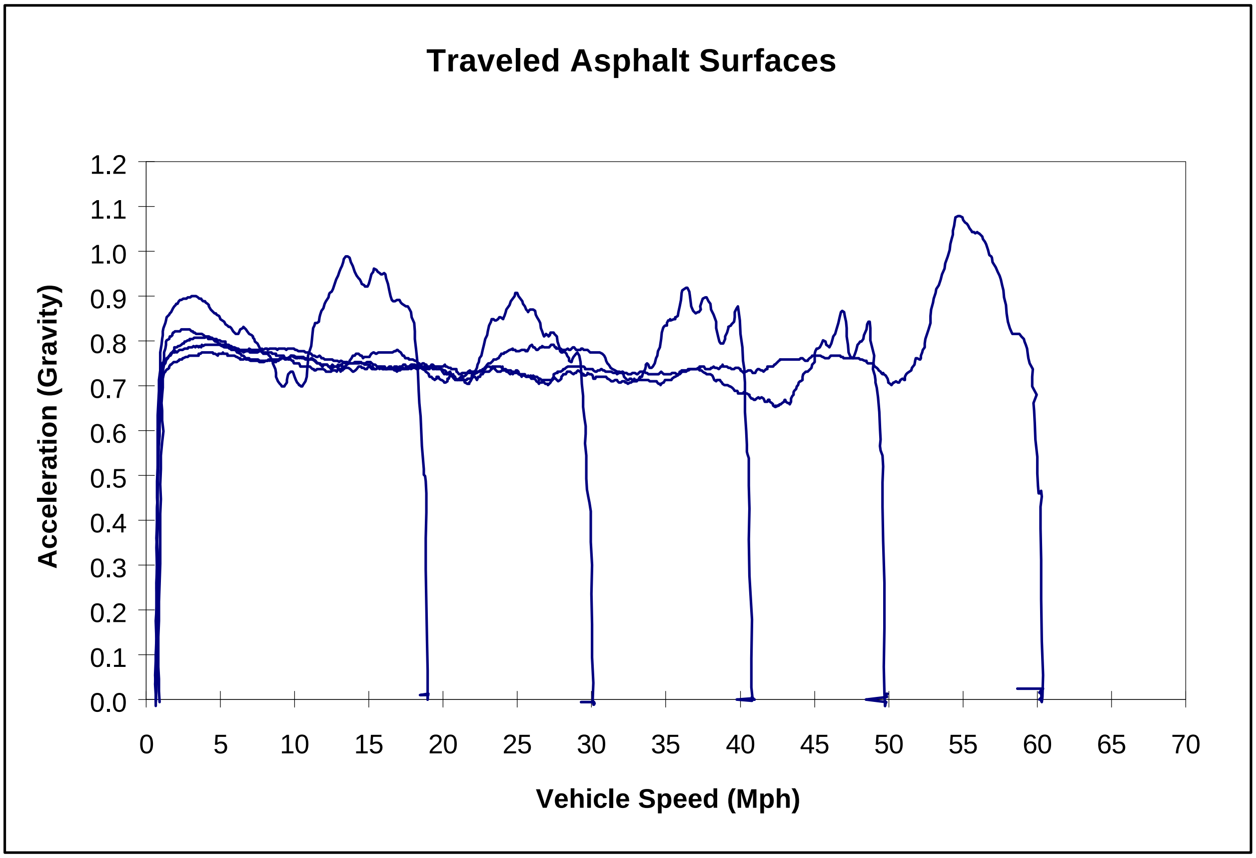

Figure A3. Characteristic Deceleration Curves for Locked Wheel Braking.

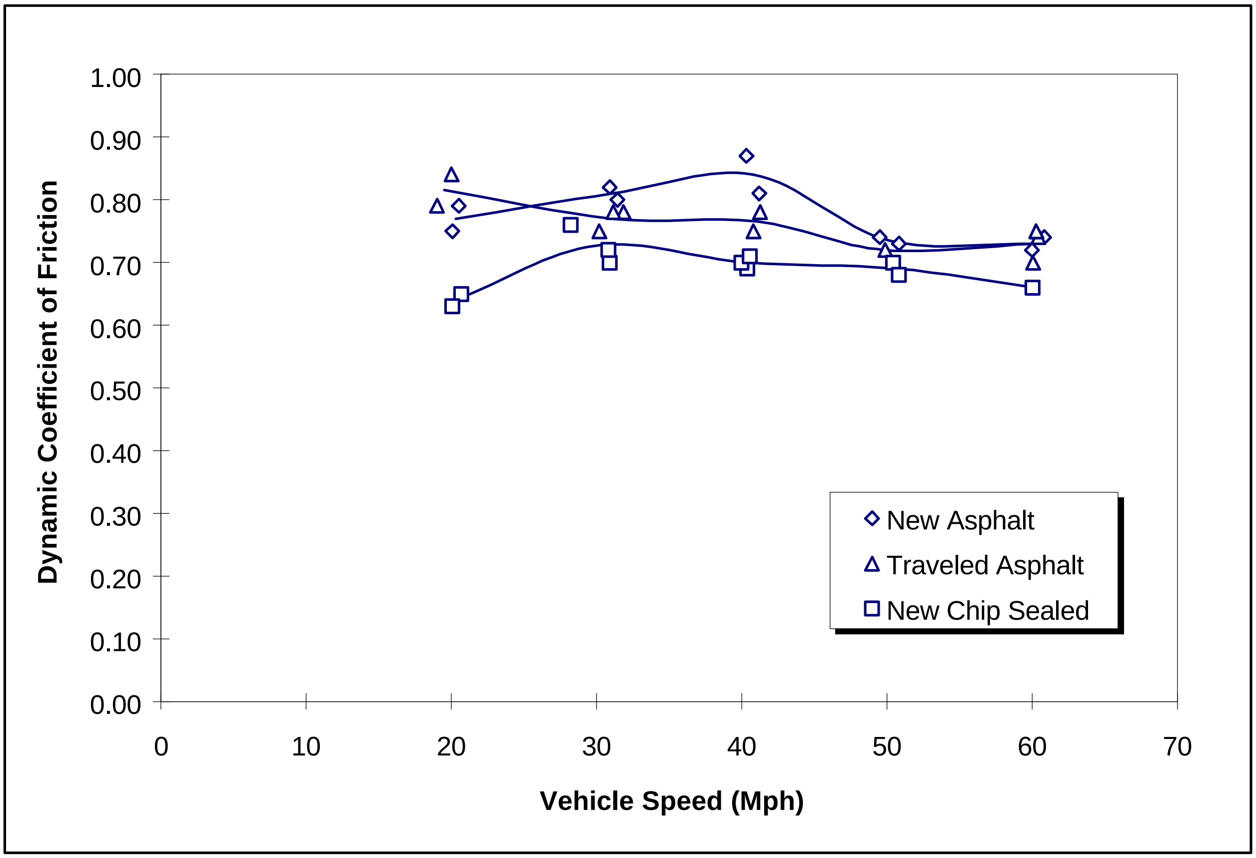

Figure A4. Dynamic Coefficient of Friction vs Vehicle Speed.

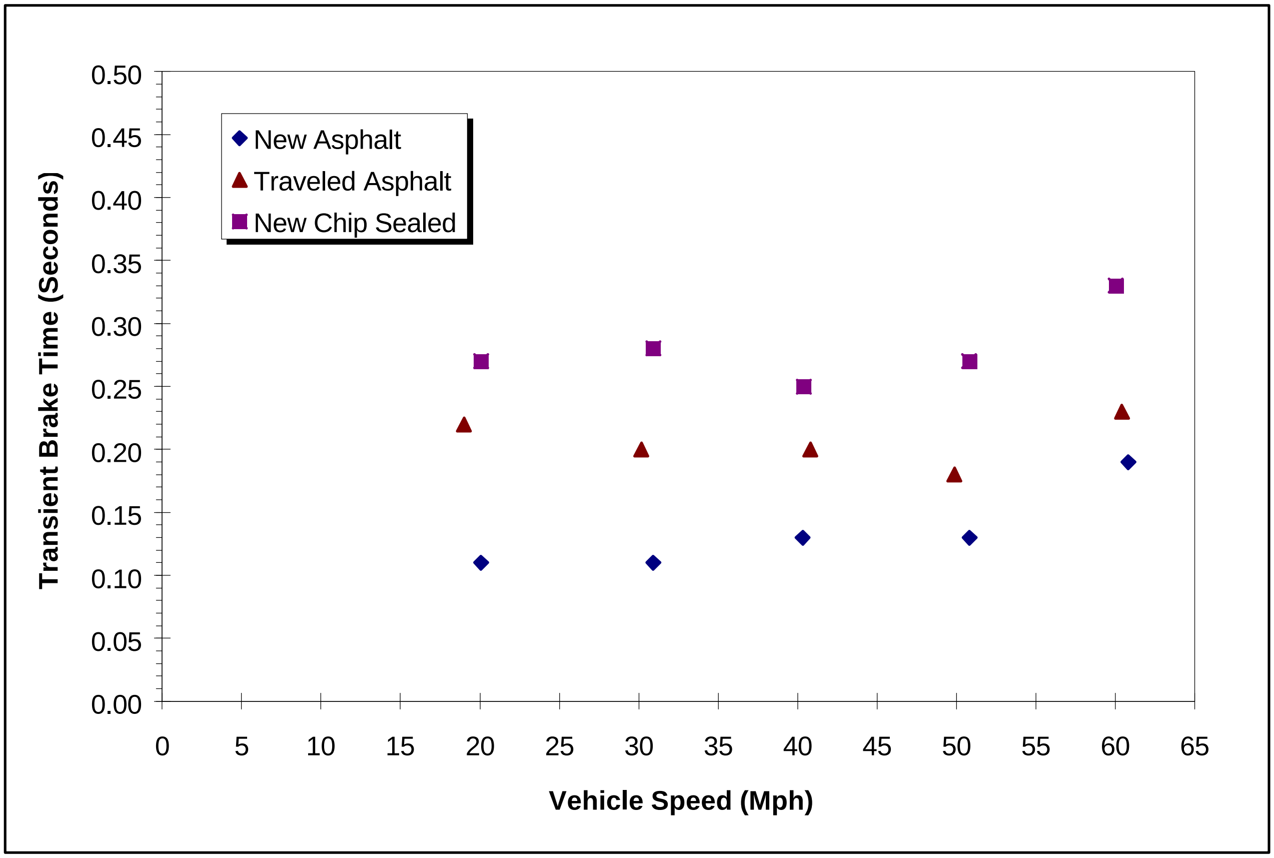

Figure A5. Minimum Transient Brake Time vs Vehicle Speed.

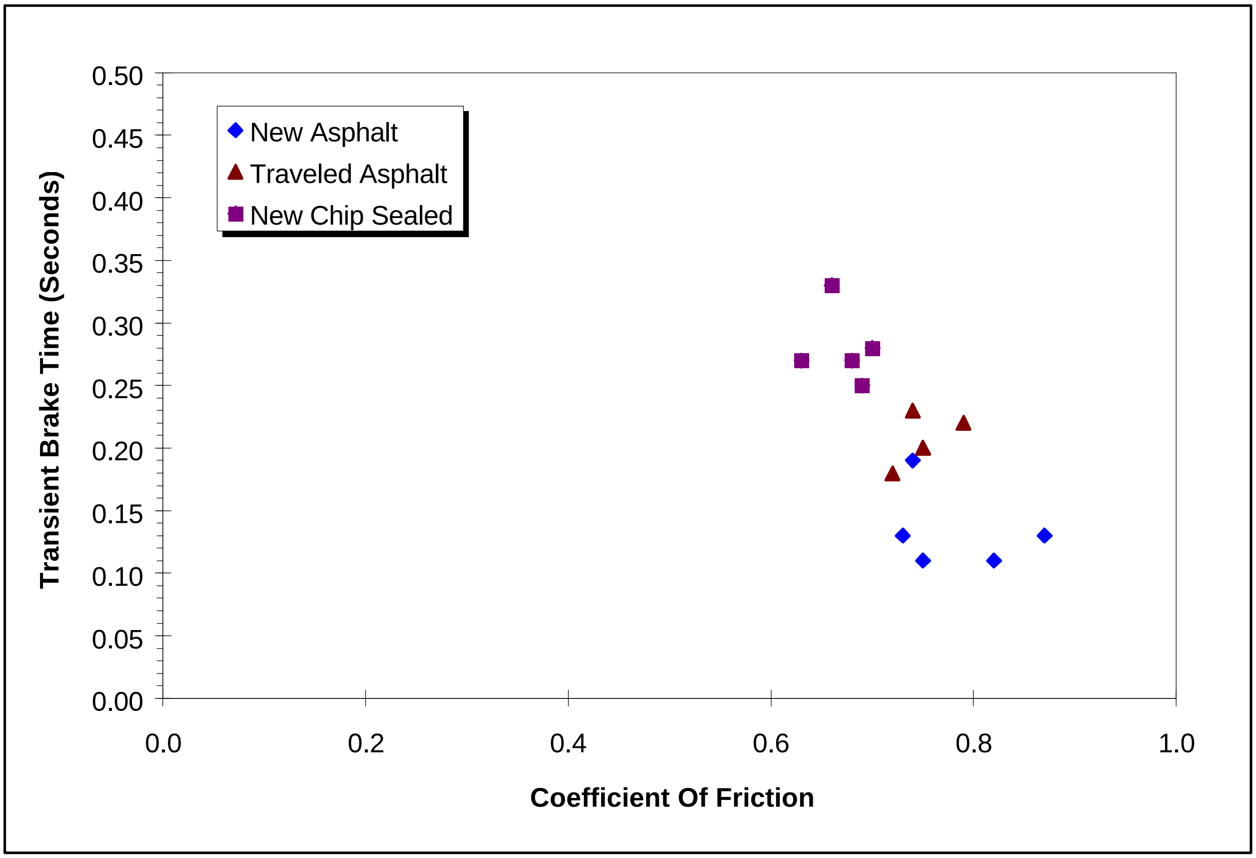

Figure A6. Minimum Transient Brake Time vs Coefficient of Friction.

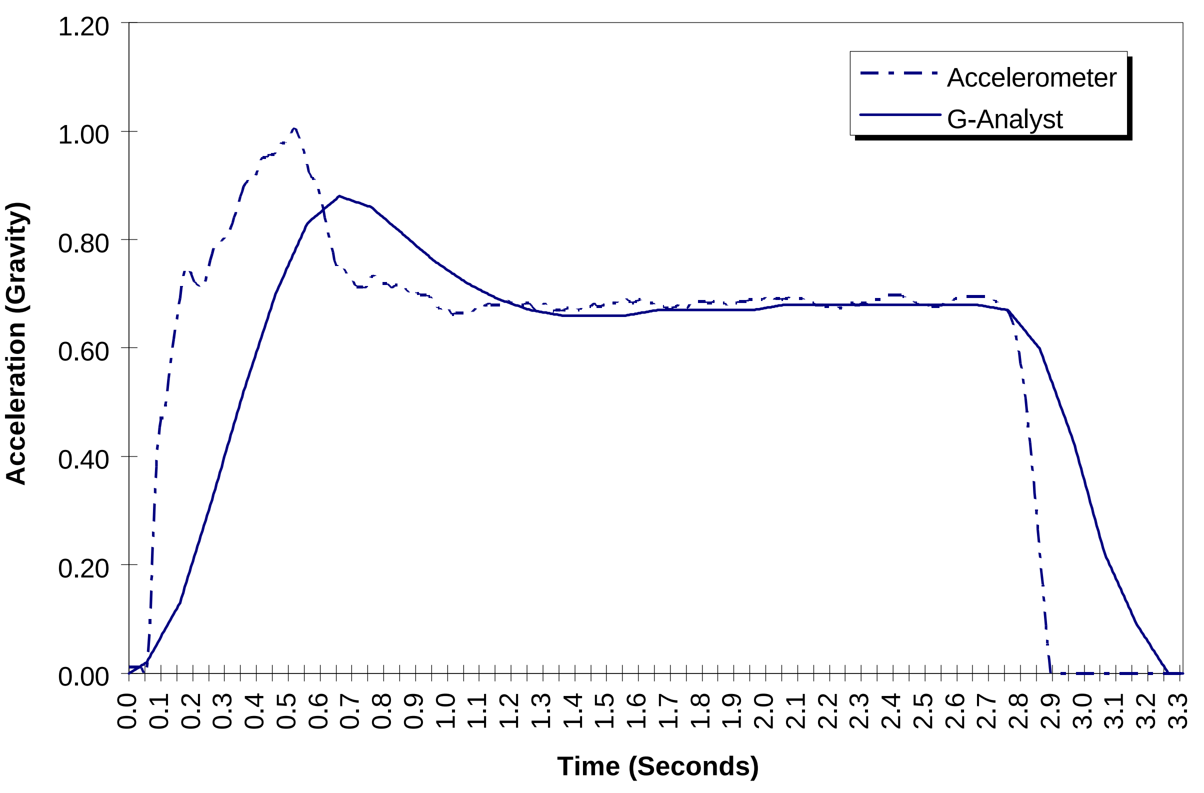

Figure A7. Acceleration vs Time.

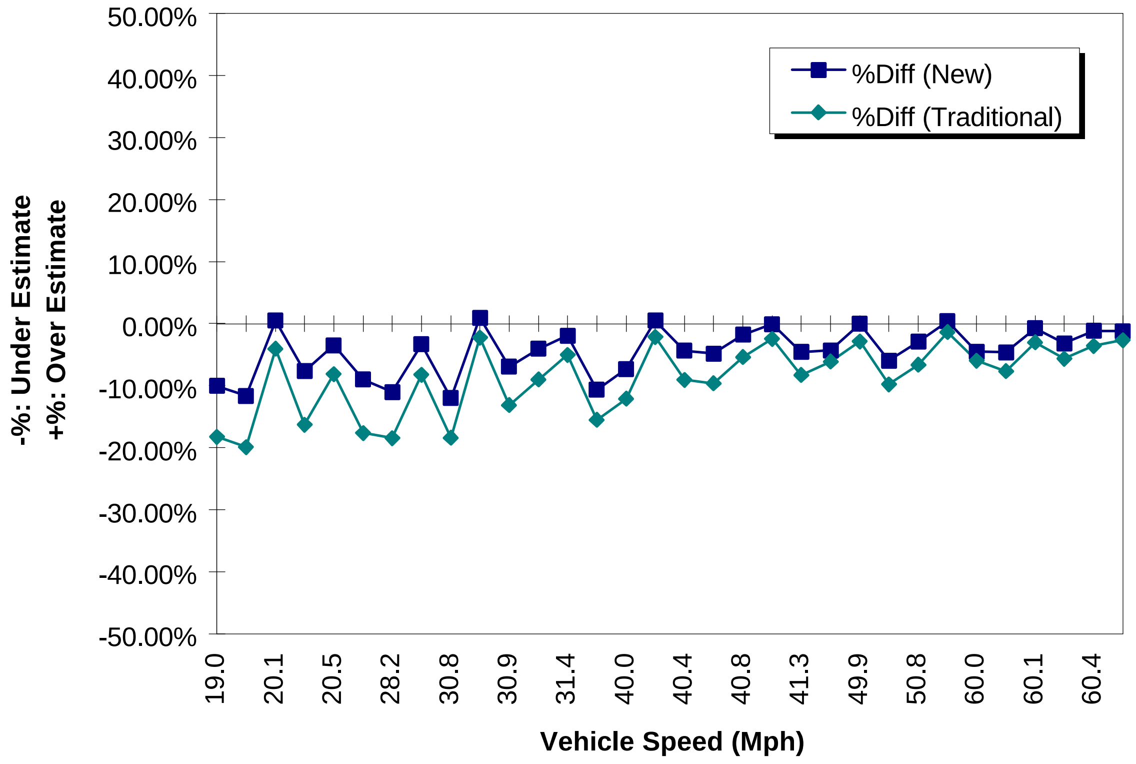

Figure A8. Accuracy of Vehicle Speed Calculations, New Method vs Traditional Method.

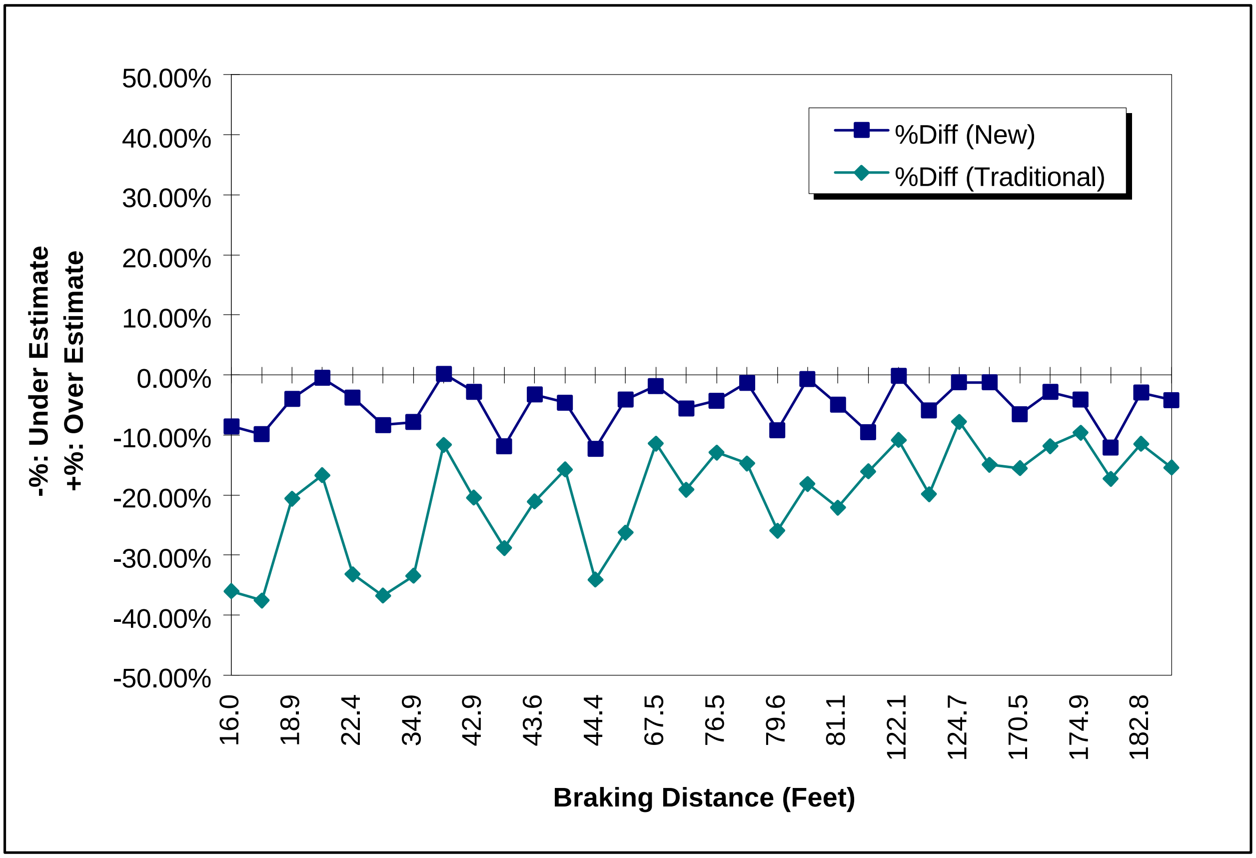

Figure A9. Accuracy of Braking Distance Calculations, New Method vs Traditional Method.

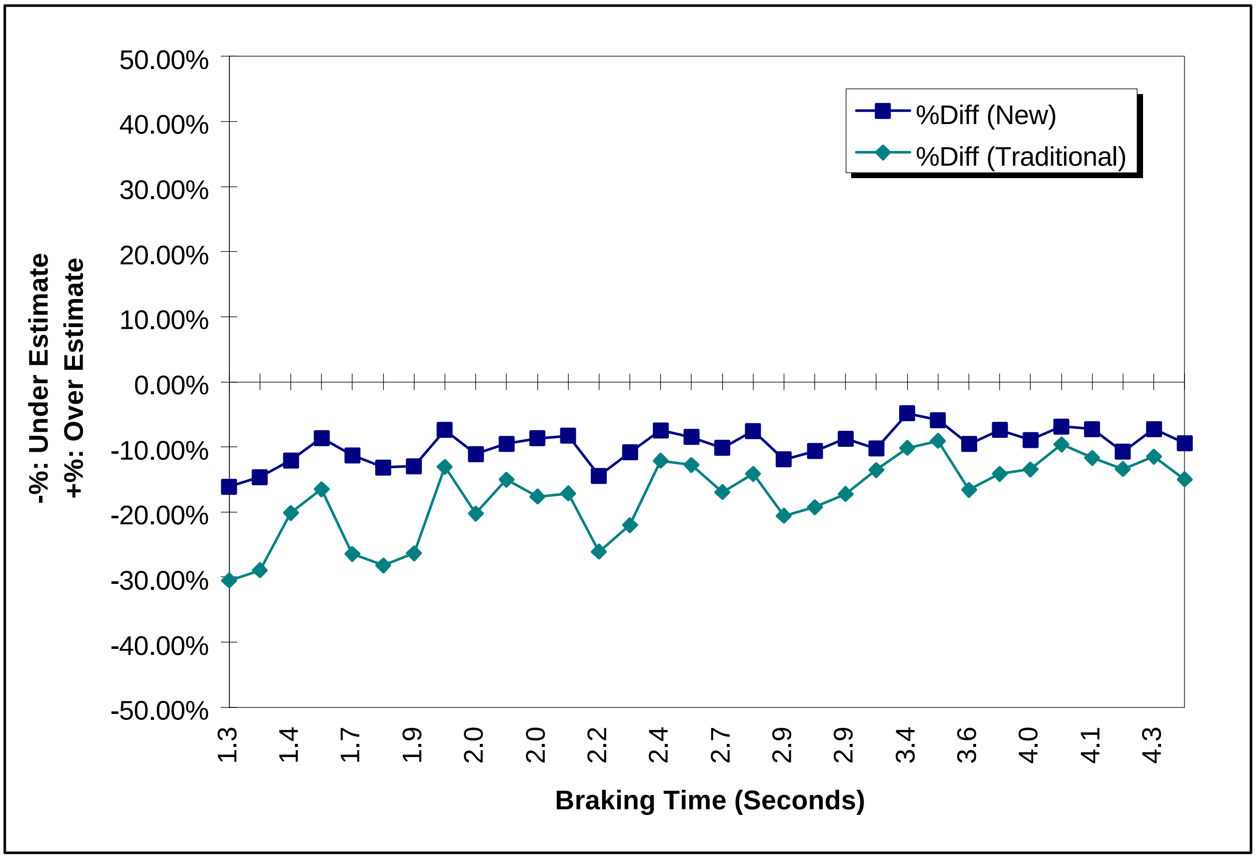

Figure A10. Accuracy of Braking Time Calculations, New Method vs Traditional Method.

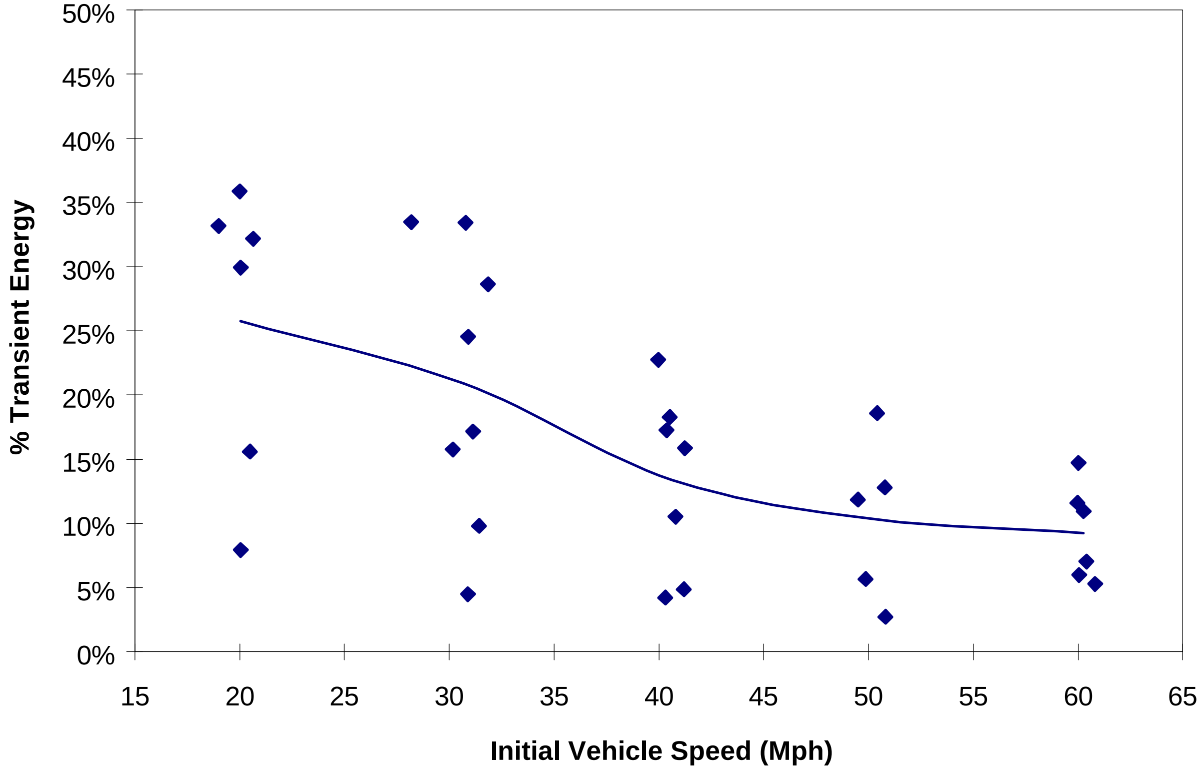

Figure A11. Transient Braking Energy as a Percentage of Initial Kinetic Energy.

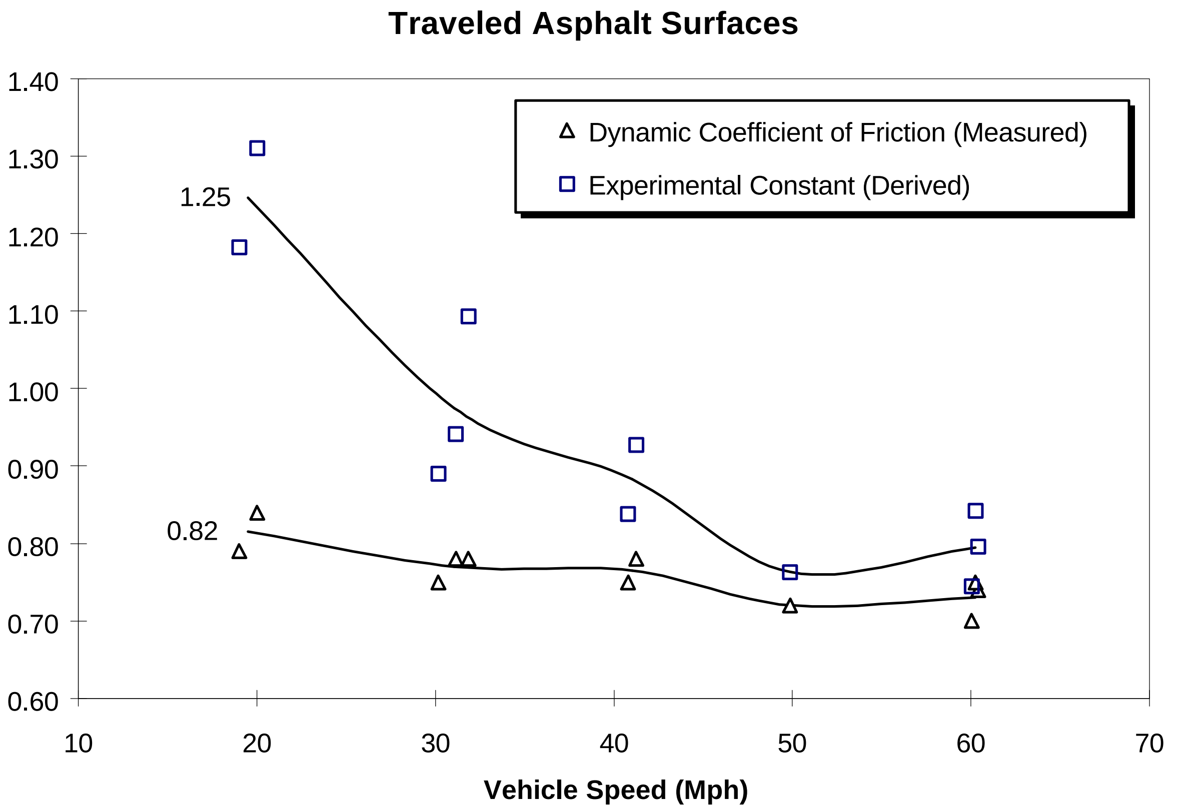

Figure A12. Dynamic Coefficient of Friction vs Vehicle Speed.

EXAMPLE AVOIDABILITY PROBLEM

Given:

Traveled asphalt roadway.

$V_{speed\_limit} := 45 \cdot mph$

$t_s := 0.18 \cdot sec$

$V_c := 37 \cdot mph$

$S_s := 59 \cdot ft$

$g := 32.2 \cdot \frac{ft}{sec^2}$

$PDR := 1.5 \cdot sec$

$\mu := 0.78$

Define functions:

Rounding function
r = value to be rounded
p = precision
u = units

$Round(r, p, u) := if(mod(r, p \cdot u) > 0.4999 \cdot p \cdot u, (r + p \cdot u) - mod(r, p \cdot u), r - mod(r, p \cdot u))$

Solution:

Traditional

New Method

Reconstruction

Speed

$V_{trad} := \sqrt{V_c^2 + 2 \cdot g \cdot \mu \cdot S_s}$

$V_{trad} := Round(V_{trad}, 1, mph)$

$V_{trad} = 52 \cdot mph$

$V_{new} := \sqrt{V_c^2 + 2 \cdot g \cdot \mu \cdot S_s} + \frac{1}{2} \cdot g \cdot \mu \cdot t_s$

$V_{new} := Round(V_{new}, 1, mph)$

$V_{new} = 54 \cdot mph$

Location at onset of braking

$S_{trad\_onset\_braking} := S_s$

$S_{trad\_onset\_braking} := Round(S_{trad\_onset\_braking}, 1, ft)$

$S_{trad\_onset\_braking} = 59 \cdot ft$

$$S_{new\_onset\_braking} := S_s + \sqrt{V_c^2 + 2 \cdot g \cdot \mu \cdot S_s} \cdot t_s \cdots + \frac{1}{3} \cdot g \cdot \mu \cdot t_s^2$$

$S_{new\_onset\_braking} := Round(S_{new\_onset\_braking}, 1, ft)$

$S_{new\_onset\_braking} = 73 \cdot ft$

Location at the onset of a PDR process of 1.5 seconds

$S_{trad\_onset\_PDR} := S_{trad\_onset\_braking} \cdots + V_{trad} \cdot PDR$

$S_{trad\_onset\_PDR} := Round(S_{trad\_onset\_PDR}, 1, ft)$

$S_{trad\_onset\_PDR} = 173 \cdot ft$

$S_{new\_onset\_PDR} := S_{new\_onset\_braking} \cdots + V_{new} \cdot PDR$

$S_{new\_onset\_PDR} := Round(S_{new\_onset\_PDR}, 1, ft)$

$S_{new\_onset\_PDR} = 192 \cdot ft$

Avoidability at Speed Limit

Stopping distance

$$S_{\text{trad\_skid}} := \frac{V_{\text{speed\_limit}}^{2}}{2 \cdot g \cdot \mu}$$

$$S_{\text{trad\_skid}} := \text{Round}\left(S_{\text{trad\_skid}}, 1, \text{ft}\right)$$

$$S_{\text{trad\_skid}} = 87 \cdot \text{ft}$$

$$S_{\text{new\_skid}} := \frac{\left(V_{\text{speed\_limit}} - \frac{1}{2} \cdot g \cdot \mu \cdot t_{s}\right)^{2}}{2 \cdot g \cdot \mu}$$

$$S_{\text{new\_skid}} := \text{Round}\left(S_{\text{new\_skid}}, 1, \text{ft}\right)$$

$$S_{\text{new\_skid}} = 81 \cdot \text{ft}$$

$$S_{\text{new\_brake}} := \left(S_{\text{new\_skid}} + \sqrt{2 \cdot g \cdot \mu \cdot S_{\text{new\_skid}}} \cdot t_{s} \cdots + \frac{1}{3} \cdot g \cdot \mu \cdot t_{s}^{2}\right)$$

$$S_{\text{new\_brake}} := \text{Round}\left(S_{\text{new\_brake}}, 1, \text{ft}\right)$$

$$S_{\text{new\_brake}} = 93 \cdot \text{ft}$$

$$S_{\text{trad\_stop}} := V_{\text{speed\_limit}} \cdot \text{PDR} \cdots + S_{\text{trad\_skid}}$$

$$S_{\text{trad\_stop}} := \text{Round}\left(S_{\text{trad\_stop}}, 1, \text{ft}\right)$$

$$S_{\text{trad\_stop}} = 186 \cdot \text{ft}$$

$$S_{\text{new\_stop}} := V_{\text{speed\_limit}} \cdot \text{PDR} \cdots + S_{\text{new\_brake}}$$

$$S_{\text{new\_stop}} := \text{Round}\left(S_{\text{new\_stop}}, 1, \text{ft}\right)$$

$$S_{\text{new\_stop}} = 192 \cdot \text{ft}$$

Stopping point relative to the POI

$$\Delta S_{\text{trad}} := S_{\text{trad\_stop}} - S_{\text{trad\_onset\_PDR}}$$

$$\Delta S_{\text{trad}} = 13 \cdot \text{ft}$$

Beyond the POI

NOT AVOIDABLE

$$\Delta S_{\text{new}} := S_{\text{new\_stop}} - S_{\text{new\_onset\_PDR}}$$

$$\Delta S_{\text{new}} = 0 \cdot \text{ft}$$

At the POI

AVOIDABLE