ABSTRACT

CRASH3 based computer programs model a vehicle structure as a homogeneous body. Crush stiffness coefficients determined from full-overlap crash tests, when used in these computer programs allow for an accurate reconstruction of collisions where the accident damage profiles are full-overlap. The structures of vehicles, however, might not be purely homogeneous in their crush response. How accurately do crush stiffness coefficients that were determined from full-overlap crash tests represent the crush response of that same vehicle in a partial-overlap/offset frontal collision? Before this question can be answered a method needs to be developed for determining crush stiffness coefficients from partial-overlap/offset frontal test collisions. These crush stiffness coefficients then could be used in a comparative analysis of the crush response of vehicles tested in both full-overlap and partial-overlap/offset frontal collisions.

A method is set forth that allows for the determination of crush stiffness coefficients from tests involving partial-overlap/offset frontal collisions with a fixed deformable barrier. This method is extended to tests involving side impacts with movable barriers.

INTRODUCTION

Vehicular structures are modeled in the CRASH3 damage algorithm as being homogeneous with respect to their stiffness characteristics. Vehicles are divided into three structures (front, rear and side). Each portion of a vehicular structure is assumed to have the same stiffness characteristics as any other portion of the same structure. This model has been generally accepted as being a reasonable approximation. Crush stiffness coefficients determined from the widely available full-overlap frontal crash tests are good predictors of the crush response of these vehicles in full-overlap collisions [see Appendix for definitions].

Vehicle structures, however, are constructed of many sub-structures and may not be purely homogeneous with respect to their stiffness characteristics. For example, in a side impact a wheel and suspension assembly has stiffness characteristics that are greatly different from that of a door structure. Crush stiffness coefficients determined from the widely available full-overlap frontal crash tests might not be good predictors of crush response for the same vehicle in a real-world accident when the damage is partial-overlap/offset.

In the past, the lack of partial-overlap/offset frontal collision tests meant that engineers/reconstructionists only had crush stiffness coefficients available that were determined from full-overlap frontal crash tests. The issue of the degree to which the front-end structure of vehicles behaved as homogeneous bodies was academic.

Recently data from partial-overlap/offset frontal collisions have become available. The number of partial-overlap/offset tests, however, is few and most vehicles probably will never be tested. Therefore, a need exists for an understanding of the degree to which vehicle frontal structures behave in a homogeneous manner in a collision. These new crash tests should provide the basis for a comparative analysis of full-overlap crush response versus partial-overlap/offset crush response. Before this analysis can be performed, however, a method needs to be developed for determining crush stiffness coefficients from partial-overlap/offset frontal test collisions.

A method is set forth in this paper that can be used to determine crush stiffness coefficients from partial-overlap/offset frontal test collisions. This method also can be used to determine stiffness coefficients from side impact tests involving moving barriers.
METHOD

There are two modules to the method. One module uses the crush stiffness determination method from reference [1]. The second module uses the impact analysis/reconstruction method from reference [2]. In the first module, the direct damages sustained in a crash test by the test vehicle and the deformable barrier face are analyzed to determine stiffness coefficients for the test vehicle. These stiffness coefficients are then used in the second module to reconstruct the test collision for the known impact speed. When properly performed with accurate test data, the reconstructed impact speed should closely match the measured impact speed for the test. Adjustments in the barrier stiffness used in the first module might be required such that the determined stiffness coefficients for the test vehicle, when used in the second module, will result in an exact match with the known test impact speed.

The stiffness module is based upon the CRASH3 damage algorithm. Intrinsic to the CRASH3 [6] damage algorithm is Newton's Third Law of Motion that states that the forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction and collinear. In the first module, the collision force exerted on the barrier is calculated based upon its known stiffness characteristics and the magnitude of its direct damage residual crush [see Appendix for sample calculations]. The collision force exerted on the test vehicle is set equal to the calculated collision force of the barrier. Next the damage onset speed [7], \( b_0 \), of the test vehicle structure is estimated. An estimation of the damage onset speed is required when mathematically determining the CRASH3 stiffness coefficients, \( A \) and \( B \). The estimated damage onset speed is then used, with the calculated collision force and the test vehicle direct damage residual crush, to determine the crush stiffness coefficients of the test vehicle. It should be noted that the direct damages on the barrier and test vehicle should have the same width. Details of this procedure can be found in reference [1] and in the Appendix.

The impact/reconstruction module is based upon the rigid body dynamics of the CRASH3 damage algorithm and analyzes the momentum exchange at impact. The condition of a common velocity occurring at the impulse center of the damage is an essential component of the method. One of the structures, either the test vehicle or the barrier, being stationary at impact is an important test condition for this module. Another test condition of importance is the moving/striking vehicle not rotating about its center of mass as it approaches the collision. This means that the impact velocity vector located at the impulse center is equal to the impact velocity vector located at the center of mass. Using this information, in conjunction with the calculated change of velocity, \( \Delta \mathbf{v}_r \), of the test vehicle, the velocity polygon located at the impulse center of the test vehicle is solved.

\[
- \Delta \mathbf{v}_r = \mathbf{v}_r - \mathbf{v}_{\text{common}}
\]

(1)

Where:

- \( \Delta \mathbf{v}_r \) = the change of velocity vector for the impulse center of the test vehicle.
- \( \mathbf{v}_{\text{common}} \) = the separation velocity vector of the impulse centers of both the test vehicle and the barrier.
- \( \mathbf{v}_r \) = the impact velocity vector for the test vehicle located at the impulse center.

For partial-overlap/offset frontal collisions, the magnitude of the common velocity vector is zero. This is due to the fact that the barrier is stationary. The impact velocity vector of the test vehicle at the impulse center, therefore, is equal in magnitude to the \( \Delta \mathbf{v}_r \) and opposite in direction.

\[
\mathbf{v}_r = -\Delta \mathbf{v}_r
\]

(2)

For the side impact tests, the solution of the vector polygon at the impulse center of the test vehicle provides a quantification of the common velocity vector, \( \mathbf{v}_{\text{common}} \). The magnitude of the impact velocity vector for the stationary test vehicle is zero. The common velocity vector, therefore, is equal to speed change, \( \Delta \mathbf{v}_r \), at the impulse center of the test vehicle.

\[
\mathbf{v}_{\text{common}} = \Delta \mathbf{v}_r
\]

(3)

The common velocity vector is used, in conjunction with the calculated \( \Delta \mathbf{v}_b \) of the moveable barrier, to solve the velocity polygon located at the impulse center of the barrier. This results in a quantification of the impact velocity vector. Details of this procedure can be found in reference [2] and in the Appendix.

\[
\mathbf{v}_b = -\Delta \mathbf{v}_b + \mathbf{v}_{\text{common}}
\]

(4)

* The numbers in the brackets refer to references listed at the end of the paper.
Where:

\[ \Delta \vec{V}_b = \text{the change of velocity vector for the impulse center of the barrier.} \]

\[ \vec{V}'_b = \text{the impact velocity vector for the barrier located at the impulse center.} \]

An example of the method has been provided in the Appendix. Data from a partial-overlap/offset frontal crash test involving a 1995 Ford Taurus was obtained from the Insurance Institute for Highway Safety [10]. The test vehicle struck a deformable fixed barrier at a speed of 64.4 km/h (40.0 mph). The edge of the barrier face was aligned to the left of the centerline of the test vehicle 16.3 cm (6.4 inch) at impact. This is a crash test alignment overlap of 41% on the driver's side. The front end of the test vehicle was displaced rearward through a maximum distance of 65 cm (25.6 inch) at the vehicle centerline and 61 cm (24.0 inch) at the left side. Direct damage extended across the front end from the left front corner toward the right to the centerline of the vehicle, indicating a direct damage overlap of 50%.

![Figure 1: 1995 Ford Taurus Collision](image1)

![Figure 2: 1995 Ford Taurus Post Collision](image2)

**DISCUSSION**

This paper would not be complete without a discussion of the potential inaccuracies associated with the application of this method. Potential inaccuracies associated with the determination of the collision forces for the barrier could be doubled. This would occur when the collision force for the test vehicle is set equal to the collision force calculated for the barrier. In other words, if the collision force calculated for the barrier is underestimated, then the collision force for the test vehicle will be underestimated by the same amount. This would result in an underestimation of the damage energies for both structures and ultimately an underestimation of the crush stiffness coefficient for the test vehicle.

The SMAC crush stiffness coefficients for the barrier face, from the available tests, have a mean value of 270 lb/in [see Appendix]. The maximum and minimum values are equal to the mean value +/- 10%. This variance in the stiffness of the barrier face could potentially introduce inaccuracies into the stiffness coefficients determined for the test vehicle.

These potential inaccuracies are essentially mitigated when module two is used to reconstruct the test crash and the reconstructed impact speed is compared to the actual test impact speed. Reasonable adjustments in the barrier face stiffness that result in a matching of the reconstructed versus actual impact speed should correct for these potential inaccuracies.

Additionally, there exists a potential for inaccuracy in the estimation of a damage onset speed for the direct damage on the test vehicle. Available crash test data can provide some insight and guidance into the estimation of a reasonable damage onset speed. Data from 2255 crash tests for vehicle model years 1960-1994 have been analyzed [9]. Low speed impacts of 15 miles per hour, or less, accounted for 439 of these tests. Damage onset speeds, \( b_0 \), were determined for the front-end structures of 126 vehicles and the rear-end structures of 52 vehicles. The distribution of the frequency of occurrence for the damage onset speed for both front and rear structures was found to be centered around speed of approximately 4.5 miles per hour.

Crush stiffness coefficients determined with this method can be considered to represent the structural characteristics of the test vehicle and, therefore, can be used as a basis for a comparative analysis of the crush response of frontal vehicular structures in full-overlap versus partial-overlap/offset collisions. These crush stiffness coefficients also can be used with sound engineering judgement in the reconstruction of traffic accidents.

**SUMMARY**

1. A need exists for an understanding of the degree that vehicle frontal structures behave in a homogeneous manner in a collision. Before a comparative analysis of full-overlap crush response versus partial-overlap/offset crush response can be
performed, a method needs to be developed for
determining crush stiffness coefficients for partial-
overlap/offset frontal test collisions.

2. A method is set forth in this paper that can be used
to determine crush stiffness coefficients from
partial-overlap/offset frontal test collisions. This
method also can be used to determine stiffness
coefficients from side impact tests involving moving
barriers.

3. The potential inaccuracies associated with this
method are essentially mitigated by reasonable
estimates of the test vehicle damage onset speed
and the use of module two. In module two, the
reconstructed impact speed is compared to the
actual test impact speed. Reasonable adjustments
in the barrier face stiffness are then made that result
in a matching of the reconstructed versus actual
impact speed.

4. Crush stiffness coefficients determined with this
method can be used as a basis for a comparative
analysis of the crush response of frontal vehicular
structures in full-overlap versus partial-overlap/offset collisions.

5. Crush stiffness coefficients determined with this
method can be used with sound engineering
judgement in the reconstruction of traffic accidents.

REFERENCES

Determining Accident Specific Crush Stiffness

2. Neptune, James A., J.E. Flynn, H.W. Underwood,
P.A. Chavez, “Impact Analysis Based Upon the
CRASH3 Damage Algorithm,” SAE Paper 950358,
1995.

3. Saha, Nripen, S. Calso, D. Midoun, P. Prasad,
“Critical Comparisons of US and European Dynamic

4. Test data of deformable barrier face material
obtained from: Plascore, Inc., Zeeland, MI.

5. NHTSA Crash Test Reports No. 114, 435 and 609.

U.S. Department of Transportation, NHTSA,
Washington D.C.

7. Campbell, K.L., "Energy Basis for Collision

8. ISO/DIS 6813 Road Vehicles - Collision
Classification - Terminology, Draft submitted to ISO
on 940607.

9. "Stiffness Coefficients for Vehicle Model Years

10. "Insurance Institute for Highway Safety (IIHS)
Crashworthiness Evaluation, Crash Test Report
(CF95013)," IIHS, Arlington, VA.

APPENDIX

Definition of Terms

Vehicle Contact Plane
A vertical plane parallel to, and inline with, the
exterior side of a vehicle. The length of the
plane is equal to the length of the corresponding
side of the vehicle.

Center of Length Projection Line
A line normal to the vehicle contact plane that
passes through the mid-point of the vehicle
contact plane.

Collision Oriented
Term used to describe a condition relating to all
the involved vehicles, objects, etc.

Vehicle Oriented
Term used to describe a condition relating to a
specific involved vehicle.

Offset
Distance measured between the center of length
projection lines of the vehicle contact planes.
The offset distance is collision oriented for non-
oblique collisions.

Direct Damage
The portion of the damage profile in which
contact occurred between the vehicle and
barrier. In other words, the vehicles surface
where that involved the primary loading by the
other vehicle or object. It does not include
regions with minor scratches, etc.

Induced Damage
The portion of the damage profile upon which
an external collision force was not applied.

Direct Damage Overlap
A measurement of the length of the direct
damage along the vehicle contact plane of the
striking/struck vehicle or object. The overlap is
related to a specific vehicle in a collision. The
overlap can be express as a distance value or
percent. A percent overlap is based upon the
overall vehicle width (W103) for front and rear
impacts, and the overall vehicle length (L103)
for side impacts.

Crash Test Alignment Overlap
A measurement of the length along the vehicle
contact plane on the vehicle, or object, that
overlaps the contact plane of the other involved
vehicle, or object, based upon the test setup
alignment.

Damage Onset Speed
A damage onset speed is the maximum speed
at impact of a vehicle in a full-overlap collision
with a non-energy absorbing fixed barrier that
will not produce any residual crushing of the
vehicle structure.
## Barrier Face Crush Stiffness Coefficients

<table>
<thead>
<tr>
<th>REF NO.</th>
<th>YR</th>
<th>MAKE</th>
<th>MODEL</th>
<th>TRAN</th>
<th>WB</th>
<th>WT</th>
<th>V-EFF</th>
<th>POOF</th>
<th>%COL</th>
<th>#C's</th>
<th>X_C</th>
<th>b0</th>
<th>b1</th>
<th>K_v</th>
<th>A</th>
<th>B</th>
<th>TEST#</th>
</tr>
</thead>
<tbody>
<tr>
<td>PlasCore</td>
<td>EECV</td>
<td>MDB</td>
<td>2994</td>
<td>2994</td>
<td>22.1</td>
<td>0</td>
<td>100%</td>
<td>1</td>
<td>8.5</td>
<td>4.5</td>
<td>2.07</td>
<td>269</td>
<td>398</td>
<td>184</td>
<td>P-ECF-01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PlasCore</td>
<td>EECV</td>
<td>MDB</td>
<td>2994</td>
<td>2994</td>
<td>21.8</td>
<td>0</td>
<td>100%</td>
<td>1</td>
<td>8.3</td>
<td>4.5</td>
<td>2.10</td>
<td>298</td>
<td>453</td>
<td>198</td>
<td>P-ECF-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PlasCore</td>
<td>NHTSA</td>
<td>MDB</td>
<td>2998</td>
<td>2998</td>
<td>21.6</td>
<td>0</td>
<td>100%</td>
<td>1</td>
<td>8.3</td>
<td>4.5</td>
<td>2.06</td>
<td>246</td>
<td>337</td>
<td>154</td>
<td>P-USS-01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PlasCore</td>
<td>NHTSA</td>
<td>MDB</td>
<td>2998</td>
<td>2998</td>
<td>21.9</td>
<td>0</td>
<td>100%</td>
<td>1</td>
<td>8.4</td>
<td>4.5</td>
<td>2.05</td>
<td>243</td>
<td>336</td>
<td>153</td>
<td>P-USS-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PlasCore</td>
<td>NHTSA</td>
<td>MDB</td>
<td>3404</td>
<td>3404</td>
<td>20.7</td>
<td>0</td>
<td>100%</td>
<td>6</td>
<td>8.0</td>
<td>4.2</td>
<td>2.07</td>
<td>280</td>
<td>362</td>
<td>178</td>
<td>A:114</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PlasCore</td>
<td>NHTSA</td>
<td>MDB</td>
<td>3000</td>
<td>3000</td>
<td>27.3</td>
<td>0</td>
<td>100%</td>
<td>6</td>
<td>10.5</td>
<td>4.5</td>
<td>2.17</td>
<td>246</td>
<td>355</td>
<td>171</td>
<td>A:435</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PlasCore</td>
<td>NHTSA</td>
<td>MDB</td>
<td>3000</td>
<td>3000</td>
<td>37.1</td>
<td>0</td>
<td>100%</td>
<td>6</td>
<td>14.2</td>
<td>4.5</td>
<td>2.30</td>
<td>249</td>
<td>377</td>
<td>192</td>
<td>A:369</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stnd Weight</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>P-USS-03</td>
</tr>
</tbody>
</table>

### Crash Plot/Smac Plot

![Crash Plot](image1)

**y = 13.14x + 28.06**

**R² = 0.99**

![Smac Plot](image2)

**y = 15.83x - 0.00**

**R² = 1.00**

4.50 Default Value For "b0"
OFFSET IMPACT SAMPLE CALCULATIONS

1995 Ford Taurus 4Dr
IIHS No. CF95013

Constants

\[
g := 32.2 \, \text{ft} \left( \frac{1 \, \text{mi}}{3600 \, \text{sec}} \right)^2 \left( \frac{12 \, \text{in}}{\text{ft}} \right)^2
\]

\[
J = J := \frac{5280}{\text{mi}} \frac{\text{hr}^2 \text{in}^2}{\text{mi}^2} \frac{1}{g \cdot 12 \, \text{in}}
\]

\[
mph \, \text{fps} := \frac{5280}{\text{mi}} \frac{\text{hr}^2}{\text{sec}} \frac{\text{hr}}{\text{mi}} \frac{\text{sec}}{\text{hr}}
\]

Number of crush measurements, \( \text{ie } C_1 \ldots C_N \)

\[
N := 6 \quad i := 0 \ldots (N - 1)
\]

Number of crush zones, \( \text{ie } (N-1) \)

\[
j := 0 \ldots (N - 2)
\]

Fixed Deformable Barrier Data

Impact speed = 0 mph

\[
C_{B_i} := \begin{array}{c}
19.2 \, \text{in} \\
17.2 \, \text{in} \\
15.2 \, \text{in} \\
13.2 \, \text{in} \\
11.2 \, \text{in} \\
9.2 \, \text{in}
\end{array}
\]

\[
\beta_i := \sum_{i=1}^{5} C_{B_i} - C_{B_0} - C_{B_5}
\]

\[
\beta_B = \frac{2}{10} \sum_{i=1}^{5} C_{B_i} - C_{B_0} - C_{B_5}
\]

\[
\beta_B = 14.2 \, \text{in}
\]

\[
L_{B_i}^j
\]

\[
L_T
\]

\[
L_T
\]

\[
L_T
\]

\[
L_T
\]

\[
L_T
\]

\[
M_B := \frac{wt_B}{g}
\]

\[
\alpha_B := PDOF_B
\]

\[
A_{B_i} := 336 \, \text{lb} \text{ in}
\]

\[
X_{FB} = 10.63 \, \text{in}
\]

\[
B_{B_i} = 152 \, \text{lb} \text{ in}^2
\]

\[
X_{RB} = 10.63 \, \text{in}
\]

Test Vehicle Data

Impact speed = 40 mph

\[
\text{OAL}_T := 192.0 \, \text{in} \quad \text{OAW}_T := 71.1 \, \text{in}
\]

\[
\text{wb}_T := 106.0 \, \text{in} \quad \text{OHF}_T := 40.3 \, \text{in}
\]

\[
\text{wt}_T := 2179 \, \text{lb} \quad \text{wt}_T := 1271 \, \text{lb}
\]

\[
\text{wt}_T := 3122 \, \text{lb}
\]

\[
\text{wt}_T := \text{wt}_F + \text{wt}_T\_	ext{Rear}
\]

\[
\text{wt}_T = 3450 \, \text{lb}
\]

\[
\text{KG}_T := 13.8
\]

\[
\text{Veh Type} \quad \text{KG}
\]

\[
\text{All} \quad 13.1
\]

\[
\text{Car} \quad 13.8
\]

\[
\text{Pickup} \quad 13.4
\]

\[
\text{Utility} \quad 12.2
\]

\[
\text{Van} \quad 12.3
\]

\[
\text{Overlap}_T = 0.50
\]

\[
\text{Zones}_T = 5
\]

\[
L_T := \frac{\text{OAW}_T \cdot \text{Overlap}_T}{\text{Zones}_T}
\]

\[
L_T := \frac{7.1}{\text{in}}
\]

\[
L_T := \frac{1}{\text{in}} \left( 1 - \text{Overlap}_T \right)
\]

\[
D_{T\_	ext{profile}} = -17.8 \, \text{in}
\]

\[
a_T := \frac{\text{wt}_T \cdot \text{wt}_T\_	ext{Rear}}{\text{wt}_T}
\]

\[
a_T := 39.1 \, \text{in}
\]

\[
PDOF_T = 0 \, \text{deg} \quad \alpha_T = PDOF_B
\]

\[
X_{FT} = a_T + \text{OHF}_T
\]

\[
X_{FT} = 79.4 \, \text{in}
\]

\[
M_T := \frac{\text{wt}_T}{g}
\]

\[
X_{RT} = \text{OAL}_T - X_{FT}
\]

\[
X_{RT} = 112.6 \, \text{in}
\]
\( \mathbf{L}_B = 10^{12} \text{lb} \cdot \text{sec}^2 \cdot \text{in} \)

\( \mathbf{I}_T = \frac{\frac{\mathbf{M}_T}{\text{ft}}}{\left(12, \text{in} \cdot \text{kg} \cdot \text{ft} \right)^2} \cdot \left(\text{OAL} \cdot T^2 + \text{OAW} \cdot T^2 \right) \)

\( I_T = 27122 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in} \)

\( \mathbf{k}_B = \sqrt{\frac{12 \cdot \text{in} \cdot T \cdot g}{\text{wt}_B}} \quad \mathbf{k}_B = 1965.7 \cdot \text{in} \)

\( \mathbf{k}_T = \sqrt{\frac{12 \cdot \text{in} \cdot T \cdot g}{\text{wt}_T}} \quad \mathbf{k}_T = 55.1 \cdot \text{in} \)

\( \mathbf{L}_B_{\text{total}} : = \sum_{j} \mathbf{L}_{Bj} \quad \mathbf{L}_B_{\text{total}} = 35.5 \cdot \text{in} \)

\( \mathbf{L}_T_{\text{total}} : = \sum_{j} \mathbf{L}_{Tj} \quad \mathbf{L}_T_{\text{total}} = 35.5 \cdot \text{in} \)

**CALCULATIONS: MODULE ONE**

Calculate the collision force for the barrier.

\[ \mathbf{F}_B := \frac{\mathbf{L}_B}{\cos(\alpha_B)} \left( A_{Bj} + \frac{\mathbf{B}_j}{2} \left( C_{Bj} + C_{B(j+1)} \right) \right) \]

\[ \mathbf{F}_B = \begin{bmatrix} 22058 \\ 19897 \\ 17735 \\ 15574 \\ 13412 \end{bmatrix} \cdot \text{lb} \]

\[ \mathbf{F}_B_{\text{total}} = 88676 \cdot \text{lb} \]

Estimate the damage offset speed for the test vehicle.

\[ b_0 := 4.5 \cdot \text{mph} \cdot \sqrt{\frac{\text{Overlap}_T}{\text{wt}_T}} \quad b_{0-T} := b_0 \cdot \sqrt{\frac{\text{wt}_T_{\text{std}}}{\text{wt}_T}} \quad b_{0-T} = 3.03 \cdot \text{mph} \]

Calculate the stiffness coefficients for the test vehicle.

SET:

\[ \mathbf{F}_{Tj} = \mathbf{F}_{ Bj} \]

\[ \beta_T := \frac{\mathbf{C}_T \cdot 0 + 2 \cdot \left( \mathbf{C}_T \cdot 1 + \mathbf{C}_T \cdot 2 + \mathbf{C}_T \cdot 3 + \mathbf{C}_T \cdot 4 \right) + \mathbf{C}_T \cdot 5}{10} \]

\[ \beta_T = 24.7 \cdot \text{in} \]

\[ b_{0-T} := \sqrt{\frac{\mathbf{b}_{0-T}^2}{2 \cdot \beta_T}} \]

\[ b_{1-T} := \frac{4 \cdot \beta_T \cdot \mathbf{F}_{\text{B total}} \cdot \cos(\alpha_T)}{J \cdot \text{wt}_T} \]

\[ b_{1-T} = 1.08 \cdot \text{mph} \]

\[ \mathbf{A}_T := \frac{J \cdot \text{wt}_T \cdot b_{0-T} \cdot b_{1-T}}{\mathbf{L}_B_{\text{total}}} \]

\[ \mathbf{B}_T := \frac{J \cdot \text{wt}_T \cdot (b_{1-T})^2}{\mathbf{L}_B_{\text{total}}} \]

\[ \mathbf{F}_{T_{\text{ck}}} := \frac{\mathbf{L}_B_{\text{total}}}{\cos(\alpha_T)} \cdot (\mathbf{A}_T + \mathbf{B}_T \cdot \beta_T) \]

\[ \mathbf{F}_{T_{\text{ck}}} = 88676 \cdot \text{lb} \]

\[ \mathbf{A}_T = 254 \cdot \text{lb} \]

\[ \mathbf{B}_T = 91 \cdot \text{lb} \]

\[ \mathbf{F}_{T_{\text{ck}}} = 88676 \cdot \text{lb} \]
CALCULATIONS: MODULE TWO

Calculate the damage energy for the barrier and the test vehicle.

\[ E_{B_j} = \frac{L_{B_j}}{12 \cdot \text{in} \cdot \cos (\alpha_{B_j})} \left[ \frac{A_{B_j}^2}{2} \left( C_{B_j} + C_{B_j+1} \right) \right] + \frac{B_{B_j}}{6} \left( C_{B_j}^2 + C_{B_j} \cdot C_{B_j+1} + (C_{B_j+1})^2 \right) + \left( \frac{A_{B_j}}{2 \cdot B_{B_j}} \right)^2 \]

\[ E_B = \begin{bmatrix} 18774 \\ 15277.8 \\ 12141.8 \\ 9366.1 \\ 6950.6 \end{bmatrix} \text{ft-lb} \]

\[ E_{B\text{\_damage}} = \sum_j E_{B_j} \]

\[ E_{B\_damage} = 62510 \cdot \text{ft-lb} \]

\[ E_{T_j} = \frac{L_{T_j}}{12 \cdot \text{in} \cdot \cos (\alpha_{T_j})} \left[ \frac{A_T^2}{2} \left( C_{T_j} + C_{T_j+1} \right) \right] + \frac{B_T}{6} \left( C_{T_j}^2 + C_{T_j} \cdot C_{T_j+1} + (C_{T_j+1})^2 \right) + \left( \frac{A_T}{2 \cdot B_T} \right)^2 \]

\[ E_T = \begin{bmatrix} 19578.8 \\ 19869.5 \\ 19869.5 \\ 20758.9 \\ 21661.1 \end{bmatrix} \text{ft-lb} \]

\[ E_{T\text{\_damage}} = \sum_j E_{T_j} \]

\[ E_{T\_damage} = 101738 \cdot \text{ft-lb} \]

\[ E_{T\_Norm} = \sqrt{\frac{2 \cdot E_{T\_damage}}{L_{T\_total}}} \]

\[ E_{T\_Norm} = 262.1 \cdot \text{lb}^{0.5} \]

\[ V_{T\_Effective} = \sqrt{\frac{2 \cdot E_{T\_damage}}{\text{wt}_T}} \]

\[ V_{T\_Effective} = 29.7 \cdot \text{mph} \]

\[ K_{V_T} = \frac{J \cdot \text{wt}_T \cdot (V_{T\_Effective})^2}{L_{B\_total} \cdot \beta_T^2} \]

Calculate the ΔV for the barrier and test vehicle.

\[ L_{T\_global_0} = 0 \cdot \text{in} \]

\[ L_{T\_global_j+1} = L_{T\_global_j} + L_{T_j} \]

\[ L_{T\_global} = \begin{bmatrix} 0 \\ 7.1 \\ 14.2 \\ 21.3 \\ 28.4 \\ 35.5 \end{bmatrix} \cdot \text{in} \]
Area $T_j := \frac{C T_j + C T_{j+1}}{2} \cdot L T_j$

$X T_j := \frac{C T_j + C T_{j+1}}{4}$

$\sum \text{(Area} \cdot X T_j) \text{)}$

$X_T := \frac{\sum \text{Area } T_j}{\text{Area } T}$

$X_T = 12.4 \text{ in}$

$\text{TEMP1}_T := X_{FT} - X_T$

$\text{TEMP1}_T = 67 \text{ in}$

Area $T_{sq} := L T_j \left( \frac{C T_j + C T_{j+1}}{2} - \frac{C T_j - C T_{j+1}}{2} \right)$

Area $T_{sq} = \begin{bmatrix} 170.6 \\ 173.5 \\ 173.5 \\ 173.5 \end{bmatrix} \text{ in}^2$

$L_{sq} := L T_j + \frac{L T_j}{2}$

$L_{sq} = \begin{bmatrix} 3.6 \\ 10.7 \\ 17.8 \\ 24.9 \end{bmatrix} \text{ in}$

Area $T_{tri} := \frac{1}{2} \cdot L T_j \cdot \left| C T_j - C T_{j+1} \right|$

Area $T_{tri} = \begin{bmatrix} 1.4 \\ 0 \\ 0 \end{bmatrix} \text{ in}^2$

$L_{tri} := L T_j + \frac{L T_j}{2}$

$L_{tri} = \begin{bmatrix} 4.7 \\ 11.8 \\ 19 \end{bmatrix} \text{ in}$

$L T_{\text{zone}} := \frac{L_{sq} \cdot L_{sq} + \text{Area } T_{tri} \cdot L_{tri}}{\text{Area } T_{sq} + \text{Area } T_{tri}}$

$L T_{\text{zone}} = \begin{bmatrix} 3.56 \\ 10.66 \\ 17.77 \\ 24.91 \\ 31.99 \end{bmatrix} \text{ in}$
\[ D_{T_{\text{impulse}}} = \frac{\sum_{j} (F_{T_j} L_{T\_zone_j})}{\sum_{j} F_{T_j}} - \frac{L_{T_{\text{total}}}}{2} \quad \text{D}_{T_{\text{impulse}}} = -1.7 \cdot \text{in} \]

\[ D_T = D_{T_{\text{profile}}} + D_{T_{\text{impulse}}} \quad \text{D}_T = -19.5 \cdot \text{in} \]

\[ H_B = \frac{\sum_{j} L_{B_j}}{2} \quad H_B = 17.8 \cdot \text{in} \]

\[ H_T = \left| D_T \cos(\alpha_T) - \text{TEMP1} \cdot T \cdot \sin(\alpha_T) \right| \quad H_T = 19.5 \cdot \text{in} \]

\[ \gamma_B = \frac{k_B^2}{k_B^2 + H_B^2} \quad \gamma_B = 1 \]

\[ \gamma_T = \frac{k_T^2}{k_T^2 + H_T^2} \quad \gamma_T = 0.889 \]

\[ \Delta V_B = \frac{2 \cdot \gamma_B (E_{B_{\text{damage}}} + E_{T_{\text{damage}}})}{3600 \cdot \text{sec} \over \text{hr}} \quad \Delta V_B = 0 \cdot \text{mph} \]

\[ \Delta V_{B_{\text{impulse}}} = \frac{\Delta V_B}{\gamma_B} \quad \Delta V_{B_{\text{impulse}}} = 0 \cdot \text{mph} \]

\[ \Delta V_T = \frac{2 \cdot \gamma_T (E_{B_{\text{damage}}} + E_{T_{\text{damage}}})}{3600 \cdot \text{sec} \over \text{hr}} \quad \Delta V_T = 35.6 \cdot \text{mph} \]

\[ \Delta V_{T_{\text{impulse}}} = \frac{\Delta V_T}{\gamma_T} \quad \Delta V_{T_{\text{impulse}}} = 40 \cdot \text{mph} \]

\[ V_T = \Delta V_{T_{\text{impulse}}} \quad V_T = 40 \cdot \text{mph} \]
SIDE IMPACT SAMPLE CALCULATIONS

1995 MBZ C220 4Dr
NHTSA No. 2225

Constants

\[
g = 32.2 \frac{\text{ft}}{\text{sec}^2} \quad J = \frac{(5280 \text{ ft/_mi})^2}{(3600 \text{ sec/hr})^2} \quad (12 \text{ in/ft})^2
\]

\[J = 0.802 \frac{\text{hr}^2 \text{in}}{\text{mi}^2} \quad \text{mph fps} = \frac{5280 \text{ ft}}{\text{mi} \text{hr}} \quad \text{fps} \text{ mph} = \frac{1}{\text{mph fps}}\]

Number of crush measurements, \( i.e \ C_1, C_N \)

\[ N = 6 \quad i = 0 .. (N - 1) \]

Number of crush zones, \( i.e \ (N-1) \)

\[ j = 0 .. (N - 2) \]

Moving Deformable Barrier Data

Impact Speed = 33 mph

\[ C_{B_i} \quad \text{wt}_B = 2990 \text{ lb} \quad \text{PDOF}_B = 27 \cdot \text{deg} \]

\[ 2.6 \cdot \text{in} \quad 1.8 \cdot \text{in} \quad 2.1 \cdot \text{in} \quad 2.5 \cdot \text{in} \quad 2.9 \cdot \text{in} \quad 5.5 \cdot \text{in} \]

\[ L_{B_i} = \beta_B = 2.7 \cdot \text{in} \]

\[ 13.2 \cdot \text{in} \quad 13.2 \cdot \text{in} \quad 13.2 \cdot \text{in} \quad 13.2 \cdot \text{in} \quad 13.2 \cdot \text{in} \]

\[ C_{B_i} - C_{B_0} - C_{B_5} = 2 - \sum_i C_{B_i} \]

\[ \beta_B = \frac{2.7 \cdot \text{in}}{10} \]

\[ a_T = \frac{\text{wt}_T \cdot \text{wt}_\text{T\_Rear}}{\text{wt}_T} \quad \text{kg } \text{T} = 13.8 \]

Test Vehicle Data

Impact Speed = 0 mph

\[ \text{OAL}_T = 176.9 \text{ in} \quad \text{OAW}_T = 67.2 \text{ in} \quad C_{T_i} = \]

\[ 7.7 \cdot \text{in} \quad 10.6 \cdot \text{in} \quad 11.9 \cdot \text{in} \quad 11.2 \cdot \text{in} \quad 11.1 \cdot \text{in} \quad 8.8 \cdot \text{in} \]

\[ \text{Veh Type KG} \]

\[ \text{All} \quad 13.1 \quad \text{Car} \quad 13.8 \quad \text{Pickup} \quad 13.4 \quad \text{Utility} \quad 12.2 \quad \text{Van} \quad 12.3 \]

For \( \text{wb} < 114 \text{ in} \)

\[ \text{D T\_profile} = a_T + 4 \cdot \text{in} - \frac{\text{wb}_T}{2} \]

\[ \text{PDOF}_T = -90 \cdot \text{deg} \quad \text{PDOF}_B \]

\[ \alpha_T = \text{PDOF}_B \quad \text{PDOF}_T = -63 \cdot \text{deg} \]

\[ M_T = \frac{\text{wt}_T}{g} \quad M_B = \frac{\text{wt}_B}{g} \]

\[ A_{B_i} = 357 \cdot \text{lb in} \quad X_{FB} = 83.2 \cdot \text{in} \quad X_{RB} = 78.8 \cdot \text{in} \]

\[ k_B = 54.01 \cdot \text{in} \quad \text{per FMVSS 587.5} \]
\[ I_B = \frac{M_B}{12 \cdot \text{ft}} \cdot (k_B)^2 \]
\[ I_B = 22573 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in} \]
\[ L_{B_{\text{total}}} = \sum_j L_{B_j} \]
\[ L_{B_{\text{total}}} = 66 \cdot \text{in} \]
\[ I_T = \frac{M_T}{(12 \cdot \text{ft}) \cdot \text{KG} \cdot \text{T}} \cdot (OAL_T^2 + OAW_T^2) \]
\[ I_T = 24424 \cdot \text{lb} \cdot \text{sec}^2 \cdot \text{in} \]
\[ k_T = \sqrt{\frac{12 \cdot \text{in} \cdot \text{lb} \cdot \text{T} \cdot g}{\text{wt} \cdot \text{T}}} \]
\[ k_T = 50.9 \cdot \text{in} \]
\[ L_{T_{\text{total}}} = \sum_j L_{T_j} \]
\[ L_{T_{\text{total}}} = 66 \cdot \text{in} \]

**CALCULATIONS: MODULE ONE**

Calculate the collision force for the barrier.

\[ F_{B_j} = \frac{L_{B_j}}{\cos \left( \alpha_B \right) \cdot \left[ A_{B_j} + \frac{B_{B_j}}{2} \cdot \left[ C_{B_j} + C_{B_{(j+1)}} \right] \right]} \]
\[ F_B = \begin{bmatrix} 10895 \\ 10258 \\ 11150 \\ 12169 \\ 15991 \end{bmatrix} \cdot \text{lb} \]
\[ F_{B_{\text{total}}} = \sum_j F_{B_j} \]
\[ F_{B_{\text{total}}} = 60462 \cdot \text{lb} \]

Estimate the damage offset speed for each crush zone on the test vehicle.

\[ b_0 = 2.0 \cdot \text{mph} \]
\[ b_{0-T} = b_0 \cdot \sqrt{\frac{\text{wt} \cdot \text{T}_{\text{std}}}{\text{wt} \cdot \text{T}}} \]
\[ b_{0-T} = 1.86 \cdot \text{mph} \]

Calculate the stiffness coefficients for the test vehicle.

Let: \( F_{T_{\text{total}}} = F_{B_{\text{total}}} \)
\( F_{T_j} = F_{B_{(N-2)-j}} \)

\[ \beta_T = \frac{C_T_0 + 2 \cdot (C_T_1 + C_T_2 + C_T_3 + C_T_4) + C_T_5}{10} \]
\[ \beta_T = 10.6 \cdot \text{in} \]

\[ b_{1-T} = \frac{-b_{0-T} + \sqrt{b_{0-T}^2 + 4 \cdot \beta_T \cdot F_{T_{\text{total}}} \cdot \cos \left( \alpha_T \right)}}{2 \cdot \beta_T} \]
\[ b_{1-T} = 1.23 \cdot \text{mph} \cdot \text{in}^{-1} \]

\[ A_T = \frac{\text{J} \cdot \text{wt} \cdot \text{T} \cdot b_{0-T} \cdot b_{1-T}}{L_{B_{\text{total}}}} \]
\[ A_T \in [102 \cdot \text{lb} \cdot \text{in}^{-1}] \]

\[ B_T = \frac{\text{J} \cdot \text{wt} \cdot (b_{1-T})^2}{L_{B_{\text{total}}}} \]
\[ B_T \in [67 \cdot \text{lb} \cdot \text{in}^{-2}] \]

\[ F_{T_{\text{ck}}} = \frac{L_{B_{\text{total}}}}{\cos \left( \alpha_T \right)} \cdot (A_T + B_T \cdot \beta_T) \]
\[ F_{T_{\text{ck}}} = 60462 \cdot \text{lb} \]
CALCULATIONS: MODULE TWO

Calculate the damage energy for both vehicles.

\[
E_{B_j} := \frac{L_{B_j}}{12 \text{ in} \cdot \cos(\alpha_{B_j})^2} \left[ \frac{A_B}{2} \left( C_{B_j} + C_{B_j+1} \right) + \frac{B_B}{6} \left[ \left( C_{B_j} \right)^2 + C_{B_j} C_{B_j+1} + \left( C_{B_j+1} \right)^2 \right] + \frac{(A_{B_j})^2}{2 \cdot B_{B_j}} \right]
\]

\[
E_B = \begin{bmatrix} 2184.7 \\ 1931.9 \\ 2283 \\ 2719.2 \\ 4760 \end{bmatrix} \cdot \text{ft} \cdot \text{lb} \\
E_{B\_damage} = \sum_j E_{B_j} \\
E_{B\_damage} = 13879 \cdot \text{ft} \cdot \text{lb}
\]

\[
E_{T_j} := \frac{L_{T_j}}{12 \text{ in} \cdot \cos(\alpha_{T_j})^2} \left[ \frac{A_T}{2} \left( C_{T_j} + C_{T_j+1} \right) + \frac{B_T}{6} \left[ \left( C_{T_j} \right)^2 + C_{T_j} C_{T_j+1} + \left( C_{T_j+1} \right)^2 \right] + \frac{(A_T)^2}{2 \cdot B_T} \right]
\]

\[
E_T = \begin{bmatrix} 5333.1 \\ 7601.6 \\ 7958.4 \\ 7476.5 \\ 6146.6 \end{bmatrix} \cdot \text{ft} \cdot \text{lb} \\
E_{T\_damage} = \sum_j E_{T_j} \\
E_{T\_damage} = 34516 \cdot \text{ft} \cdot \text{lb}
\]

\[
E_{T\_Norm} = \frac{2 \cdot E_{T\_damage}}{L_{T\_total}} \\
V_{T\_Effective} = \left( \frac{2 \cdot E_{T\_damage} \cdot g}{\text{wt}_T} \right)^{0.5} \\
E_{T\_Norm} = 112 \cdot \text{lb}^{0.5} \\
V_{T\_Effective} = 16.9 \cdot \text{mph}
\]

\[
KV_T = \frac{J \cdot \text{wt}_T \cdot (V_{T\_Effective})^2}{L_{B\_total} \cdot \beta_T^2}
\]

Calculate the ΔV for the vehicles

\[
L_{B\_global_0} := 0 \cdot \text{in} \\
L_{B\_global_{j+1}} := L_{B\_global_j} + L_{B_j} \\
L_{T\_global_0} := 0 \cdot \text{in} \\
L_{T\_global_{j+1}} := L_{T\_global_j} + L_{T_j}
\]

\[
L_{B\_global} = \begin{bmatrix} 0 \\ 13.2 \\ 26.4 \\ 39.6 \\ 52.8 \\ 66 \end{bmatrix} \cdot \text{in} \\
L_{T\_global} = \begin{bmatrix} 0 \\ 13.2 \\ 26.4 \\ 39.6 \\ 52.8 \\ 66 \end{bmatrix} \cdot \text{in}
\]

\[
C_j + C_{j+1} \quad 4 \\
L_j \quad C_j
\]
Area $B_j = \frac{C_{B_j} + C_{B_{j+1}}}{2} \cdot L_{B_j}$

Area $B = \begin{bmatrix} 29 \\ 25.7 \\ 30.4 \\ 35.6 \\ 55.4 \end{bmatrix} \cdot \text{in}^2$

Area $T_j = \frac{C_{T_j} + C_{T_{j+1}}}{2} \cdot L_{T_j}$

Area $T = \begin{bmatrix} 120.8 \\ 148.5 \\ 152.5 \\ 147.2 \\ 131.3 \end{bmatrix} \cdot \text{in}^2$

$x_{B_j} = \frac{C_{B_j} + C_{B_{j+1}}}{4}$

$x_B = \begin{bmatrix} 1.1 \\ 1 \\ 1.2 \\ 1.4 \\ 2.1 \end{bmatrix} \cdot \text{in}$

$X_B = \frac{\sum_j (\text{Area } B_j \cdot x_{B_j})}{\sum_j \text{Area } B_j}$

$X_B = 1.5 \cdot \text{in}$

$Y_{T_j} = \frac{C_{T_j} + C_{T_{j+1}}}{4}$

$Y_T = \begin{bmatrix} 4.6 \\ 5.6 \\ 5.8 \\ 5.6 \end{bmatrix} \cdot \text{in}$

$Y_T = 5.3 \cdot \text{in}$

$\text{TEMP1}_B := X_{FB} - X_B$

$\text{TEMP1}_B = 81.7 \cdot \text{in}$

$\text{TEMP1}_T := \frac{OAW_T}{2} - Y_T$

$\text{TEMP1}_T = 28.3 \cdot \text{in}$

Area $T_{sq} = L_{T_j} \left( \frac{C_{T_j} + C_{T_{j+1}}}{2} - \frac{C_{T_j} - C_{T_{j+1}}}{2} \right)$

Area $T_{sq} = \begin{bmatrix} 101.6 \\ 139.9 \\ 147.8 \\ 146.5 \\ 116.2 \end{bmatrix} \cdot \text{in}^2$

Area $B_{sq} = L_{B_j} \left( \frac{C_{B_j} + C_{B_{j+1}}}{2} - \frac{C_{B_j} - C_{B_{j+1}}}{2} \right)$

Area $B_{sq} = \begin{bmatrix} 23.8 \\ 23.8 \\ 27.7 \\ 33 \\ 38.3 \end{bmatrix} \cdot \text{in}^2$

$I_{B_{sq}} = L_{B_{global_j}} + \frac{L_{B_j}}{2}$

$I_{B_{sq}} = \begin{bmatrix} 6.6 \\ 19.8 \\ 33 \\ 46.2 \\ 59.4 \end{bmatrix} \cdot \text{in}$

$I_{T_{sq}} = L_{T_{global_j}} + \frac{L_{T_j}}{2}$

$I_{T_{sq}} = \begin{bmatrix} 6.6 \\ 19.8 \\ 33 \\ 46.2 \\ 59.4 \end{bmatrix} \cdot \text{in}$
Area \( B_{\text{tri}} \) := \frac{1}{2} \cdot L_B \cdot |C_B - C_{B+1}| \\
\text{Area } B_{\text{tri}} = 5.3 \text{ \cdot in}^2 \\
17.2 \\
\begin{pmatrix}
5.3 \\
2 \\
2.6 \\
2.6 \\
17.2
\end{pmatrix}
\text{ \cdot in}^2

\text{Area } T_{\text{tri}} := \frac{1}{2} \cdot L_T \cdot |C_T - C_{T+1}|
\text{Area } T_{\text{tri}} = 19.1 \cdot \text{in}^2 \\
8.6 \\
\begin{pmatrix}
19.1 \\
8.6 \\
4.6 \\
0.7 \\
15.2
\end{pmatrix}
\text{ \cdot in}^2

\begin{align*}
L_B & = L_B_{global} + \frac{L_B}{(C_B > C_{B+1}, 3, 3)} \\
L_B & = 8.8 \\
22 \\
35.2 \\
48.4 \\
61.6
\end{align*}

\begin{align*}
L_T & = L_T_{global} + \frac{L_T}{(C_T > C_{T+1}, 3, 3)} \\
L_T & = 44 \\
44 \\
30.8 \\
44 \\
57.2
\end{align*}

L_B_{zone} := \frac{\text{Area } B_{\text{sq}} \cdot L_{B_{sq}} \cdot \text{Area } B_{\text{tri}} \cdot L_{B_{tri}}}{\text{Area } B_{\text{sq}} + \text{Area } B_{\text{tri}}}
\begin{pmatrix}
6.2 \\
19.97 \\
33.19 \\
46.36 \\
60.08
\end{pmatrix}
\text{ \cdot in}

\begin{align*}
L_T_{zone} & := \frac{\text{Area } T_{\text{sq}} \cdot L_{T_{sq}} \cdot \text{Area } T_{\text{tri}} \cdot L_{T_{tri}}}{\text{Area } T_{\text{sq}} + \text{Area } T_{\text{tri}}}
L_T & = 32.93 \cdot \text{in} \\
32.93 \\
19.93 \\
46.19 \\
59.15
\end{align*}

\begin{equation}
D_B := \sum_j \frac{\left(D_B \cdot L_{B_{zone}} \cdot L_B_{total}\right)}{2} \\
D_B := 2.8 \cdot \text{in}
\end{equation}

\begin{equation}
D_B := D_B_{impulse} \\
D_B = 2.8 \cdot \text{in}
\end{equation}

\begin{equation}
H_B := \left| D_B \cdot \cos(\alpha_B) - \text{TEMP1} \cdot \sin(\alpha_B) \right| \\
H_B = 34.6 \cdot \text{in}
\end{equation}

\begin{equation}
\gamma_B := \frac{k_B^2}{k_B^2 + H_B^2} \\
\gamma_B = 0.709
\end{equation}

\begin{equation}
\gamma_T := \frac{k_T^2}{k_T^2 + H_T^2} \\
\gamma_T = 0.976
\end{equation}

\begin{align*}
D_T & := D_T_{profile} + D_T_{impulse} \\
D_T & = 5.4 \cdot \text{in}
\end{align*}

\begin{equation}
H_T := \sqrt{D_T^2 + (\text{TEMP1} \cdot T^2) \cdot \cos \left( \arctan \left( \frac{\text{TEMP1} \cdot T}{D_T} + \alpha_T \right) \right)} \\
H_T = 8 \cdot \text{in}
\end{equation}

\begin{equation}
\gamma_T := \frac{k_T^2}{k_T^2 + H_T^2} \\
\gamma_T = 0.976
\end{equation}
\[ \Delta V_B = \frac{2 \cdot \gamma \cdot B \cdot (E_{B\_damage} + E_{T\_damage})}{M_B \left(1 + \frac{\gamma B \cdot M_B}{\gamma T \cdot M_T}\right)} \cdot \frac{3600 \text{ sec}}{hr} \cdot \frac{1}{5280 \text{ ft}} \cdot \frac{1}{\text{mi}} \]

\[ \Delta V_B = 14.7 \cdot \text{mph} \]

\[ \Delta V_{B\_impulse} = \frac{\Delta V_B}{\gamma B} \]

\[ \Delta V_{T\_impulse} = \frac{\Delta V_T}{\gamma T} \]

\[ \Delta V_T = 12.1 \cdot \text{mph} \]

\[ \Delta V_{T\_impulse} = 12.4 \cdot \text{mph} \]

Solve the vector polygon located at the impulse center of the test vehicle.

Note: This solution is relative to the test vehicle coordinate system.

SET:

\[ V_T := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ mph} \]

\[ \phi_T := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ deg sec} \]

\[ \psi_T := (\text{PDOF}_B - \text{PDOF}_T) + 180 \cdot \text{deg} \]

\[ \psi_T = 270 \cdot \text{deg} \]

\[ \text{PDOF}_B := \text{PDOF}_B \]

\[ E_B := E_{B\_damage} \]

\[ \Delta V_B := \begin{pmatrix} -\Delta V_B \cdot \cos(\text{PDOF}_B) \\ -\Delta V_B \cdot \sin(\text{PDOF}_B) \\ 0 \cdot \text{mph} \end{pmatrix} \]

\[ \Delta V_B = \begin{pmatrix} -13.07 \\ -6.66 \\ 0 \end{pmatrix} \cdot \text{mph} \]

\[ \Delta V_T := \begin{pmatrix} -\Delta V_T \cdot \cos(\text{PDOF}_T) \\ -\Delta V_T \cdot \sin(\text{PDOF}_T) \\ 0 \cdot \text{mph} \end{pmatrix} \]

\[ \Delta V_T = \begin{pmatrix} 5.47 \\ 10.74 \\ 0 \end{pmatrix} \cdot \text{mph} \]

\[ \Delta V_{B\_P} = \begin{pmatrix} \left(1 + \frac{H_B^2}{k_B^2}\right) \Delta V_{B_0} \\ \left(1 + \frac{H_B^2}{k_B^2}\right) \Delta V_{B_1} \\ 0 \cdot \text{mph} \end{pmatrix} \]

\[ \Delta V_{B\_P} = \begin{pmatrix} -18.42 \\ -9.39 \\ 0 \end{pmatrix} \cdot \text{mph} \quad | \Delta V_{B\_P} | = 20.68 \cdot \text{mph} \]

\[ \Delta V_{T\_P} = \begin{pmatrix} \left(1 + \frac{H_T^2}{k_T^2}\right) \Delta V_{T_0} \\ \left(1 + \frac{H_T^2}{k_T^2}\right) \Delta V_{T_1} \\ 0 \cdot \text{mph} \end{pmatrix} \]

\[ \Delta V_{T\_P} = \begin{pmatrix} 5.61 \\ 11.01 \\ 0 \end{pmatrix} \cdot \text{mph} \quad | \Delta V_{T\_P} | = 12.36 \cdot \text{mph} \]
\[ V_{\text{common}} = \Delta V_{T-p} \]
\[ V_{\text{common}} = \begin{pmatrix} -5.6 \\ 11 \\ 0 \end{pmatrix} \text{ mph} \]
\[ |V_{\text{common}}| = 12.4 \text{ mph} \]
\[ \theta_{\text{common}} = \angle (V_{\text{common}_0}, V_{\text{common}_1}) \]
\[ \theta_{\text{common}} = 117 \text{ deg} \]

Transform the common velocity vector from the test vehicle coordinate system to the global coordinate system.

\[ V_{\text{Common}} = \begin{pmatrix} V_{\text{common}_0} \cdot \cos(-\psi_T) + V_{\text{common}_1} \cdot \sin(-\psi_T) \\ -V_{\text{common}_0} \cdot \sin(-\psi_T) + V_{\text{common}_1} \cdot \cos(-\psi_T) \\ 0 \cdot \text{mph} \end{pmatrix} \]
\[ V_{\text{Common}} = \begin{pmatrix} 11 \\ 0 \end{pmatrix} \text{ mph} \]
\[ |V_{\text{Common}}| = 12.4 \text{ mph} \]
\[ \psi_{\text{Common}} = \angle (V_{\text{Common}_0}, V_{\text{Common}_1}) \]
\[ \psi_{\text{Common}} = 27 \text{ deg} \]

Solve the vector polygon located at the impulse center of the barrier.

\[ V_{B-p} = \left[ -(\Delta V_{B-p}) + V_{\text{Common}} \right] \]
\[ V_{B-p} = \begin{pmatrix} 29.4 \\ 15 \\ 0 \end{pmatrix} \text{ mph} \]
\[ |V_{B-p}| = 33 \text{ mph} \]
\[ \psi_{B-p} = \angle (V_{B-p}_0, V_{B-p}_1) \]
\[ \psi_{B-p} = 27 \text{ deg} \]