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Crush Stiffness Coefficients, Restitution Constants, And a Revision of CRASH3 & SMAC

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ABSTRACT

A revision of the modeling of restitution properties in the SMAC computer program and the introduction of restitution properties in the CRASH3 program have been proposed by McHenry [1]*, the author of M-CRASH and M-SMAC [2]. The accuracy of an accident reconstruction, which uses the proposed restitution model, is directly related to the accuracy of the crush stiffness coefficients employed. The ideal condition for determining crush stiffness coefficients for these programs requires crash tests of a vehicle structure through a wide range of impact speeds with a rigid barrier. Reports from these crash tests should contain the usual data found in NHTSA crash test reports plus rebound velocity and maximum dynamic crush. Unfortunately, such comprehensive tests are very rare. As a result, certain vehicle properties need to be generalized from the existing available crash tests. These generalized properties can then be used to calculate crush stiffness coefficients for these programs.

The revised restitution model extends the original CRASH3 program by accounting for vehicle restitution properties and revises the restitution model of the original SMAC program. The two different force-deflection models used in the original programs are unified into a single force-deflection model. As a result, the revised CRASH3/SMAC would use identical crush stiffness coefficients and restitution constants to describe the crush response characteristics of vehicles.

The restitution portion of the new force-deflection model has an underlying problem. The new force-deflection model coupled with improperly determined crush stiffness coefficients and restitution constants can create vehicle properties that are not realistic. Properly determined coefficients/constants, however, used in conjunction with the revised CRASH3/SMAC force-deflection model should result in improved accuracy over the original CRASH3 and SMAC programs when estimating collision severity.

Set forth in this paper is a method for determining crush stiffness coefficients and restitution constants for the revised CRASH3 and SMAC. Steps are taken to prevent unrealistic modeling of vehicle properties. Finally, this method will provide a means to calculate the coefficients/constants in the absence of comprehensive testing.

INTRODUCTION

The revised CRASH3 and SMAC would use a linear force-deflection model to represent the entire crush response of a vehicle. The model is applicable from the onset of dynamic crush to the end of the rebound process where separation occurs. The crush stiffness coefficient, K_1, represents the linear spring constant for the increasing loads of the approach phase of a collision. At the end of the approach phase, the dynamic crush, δ_m, and collision force are at a maximum. Next the collision force reduces until the vehicle structure begins the dimensional recovery that occurs during the rebound phase. The coefficient, K_2, is the linear spring constant for the decreasing loads of the rebound phase. At the end of the rebound phase, the dimensional recovery is complete. If the magnitude of the collision is sufficient, the dimensional recover is partial and the remaining damage is residual crush, δ_f. Lessor magnitude collisions will result in full dimensional recovery with no resulting residual crush.

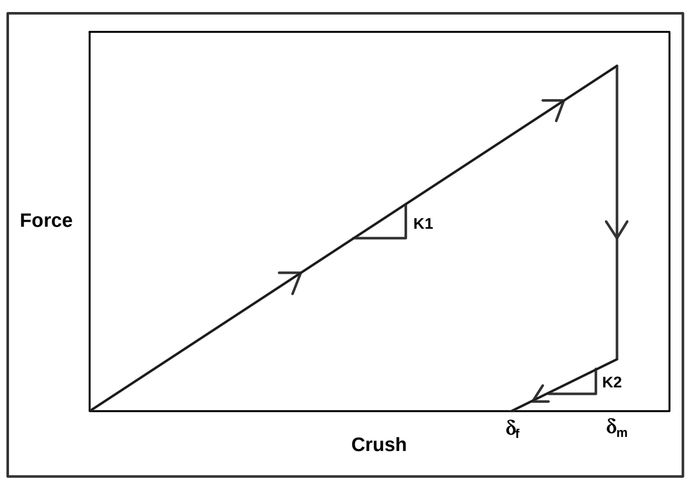

Figure 1: Force-Deflection Graph

* The numbers in the brackets refer to references listed at the end of the paper.

The revised restitution model uses an equation for the coefficient of restitution that is a function of dynamic crush and the restitution constants.

$$\varepsilon = \frac{\Gamma}{\delta_m} + \rho \qquad (1)$$

Where:

ε = Coefficient of restitution.
δ_m = Maximum dynamic crush.
Γ = Restitution constant.
ρ = Restitution constant.

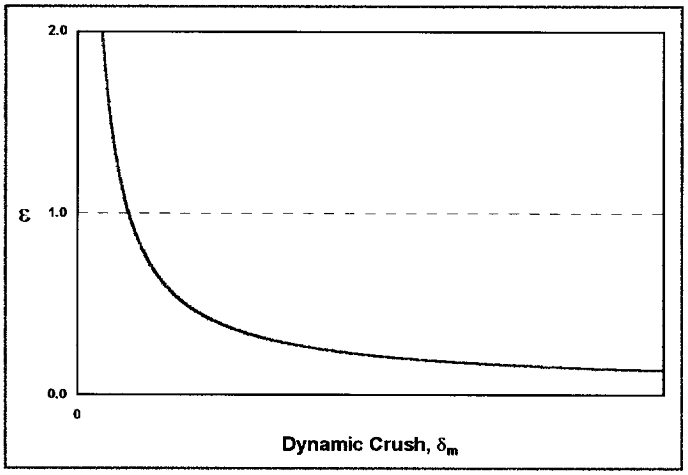

Figure 2: Restitution Equation

The revised CRASH3/SMAC also would use an equation for restitution that recognizes that the coefficient of restitution is equal to the square root of the ratio of the restored energy, E_R, over the absorbed energy, E_A. This equation describes the coefficient of restitution as a function of both residual and dynamic crush, and the revised CRASH3/SMAC stiffness coefficients, K_1 and K_2.

$$\varepsilon = \sqrt{\frac{E_R}{E_A}} = \sqrt{\frac{K_2(\delta_m - \delta_f)^2}{K_1 \delta_m^2}} = \left(1 - \frac{\delta_f}{\delta_m}\right)\sqrt{\frac{K_2}{K_1}} \qquad (2)$$

Where:

δ_f = Residual crush.
δ_m = Maximum dynamic crush.
K_1 = Stiffness coefficient for increasing loads.
K_2 = Stiffness coefficient for decreasing loads.

For the full dimensional recovery collision where residual crush does not occur ($\delta_f = 0$), equation (2) reduces to a constant.

$$\varepsilon = \sqrt{\frac{E_R}{E_A}} = \sqrt{\frac{K_2}{K_1}} \qquad (3)$$

Restitution reportedly is defined in the revised CRASH3/SMAC by equations (1) or (2) except when the equations result in a value greater than 1.0. Under this condition, the computer program code reportedly overrides these equations and sets the coefficient of restitution equal to 1.0. As a result, from the onset of dynamic crush to the point where these equations indicate a value less than 1.0, the coefficient of restitution is treated as a constant with a magnitude of 1.0.

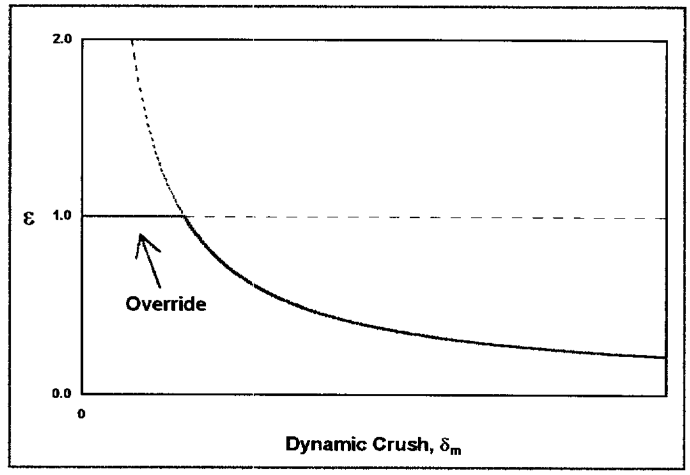

Figure 3: Restitution Equation Override For ε>1.0

A problem occurs when the revised CRASH3/SMAC sets the magnitude of the coefficient of restitution equal to 1.0. In the real world, the coefficient of restitution may approach the theoretical magnitude of 1.0 but will never equal 1.0. A perfectly elastic vehicle structure simply does not exist in the real world. Therefore, setting $\varepsilon = 1.0$ specifies properties for a vehicle that are not realistic.

Other problems occur when the loading stiffness coefficient, K_1, that is entered into the revised CRASH3/SMAC is less than the entered unloading stiffness coefficient, K_2. It can be seen in figure 4 that, for a full dimensional recovery collision, the loading stiffness must be greater than the unloading stiffness. If $K_1 < K_2$, then in a full dimensional recovery collision, the absorbed energy, E_A (area under the K_1 line), would be less than the restored energy, E_R (area under the K_2 line). This means that energy would be created in a collision. This is in violation of the law of thermodynamics regarding the conservation of energy. Therefore, it is not realistic for $K_1 < K_2$ in full dimensional recovery collisions ($\delta_f = 0$).

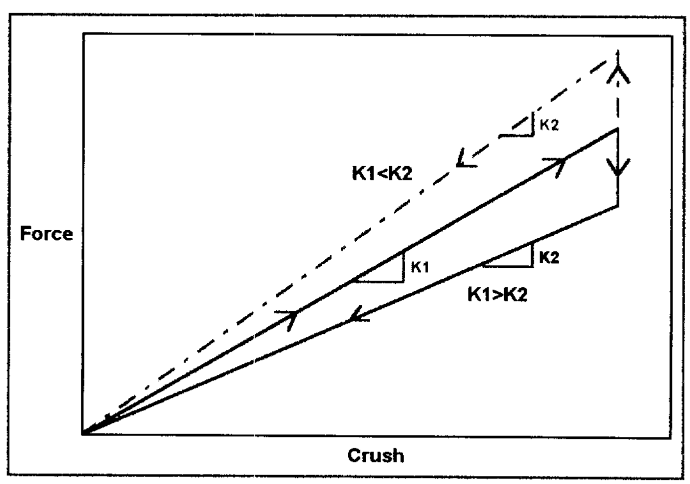

Figure 4: Full Dimensional Recovery

For partial dimensional recovery collisions, at the level of dynamic crush, δ_{m0}, residual crush begins to occur. With K_1 less than K_2, equation (2) initially would indicate a magnitude for the coefficient of restitution that is greater than 1.0 and the programs reportedly override the equation setting the magnitude equal to 1.0. This results in a condition where residual crush is occurring and at the same time the coefficient of restitution is being set equal to 1.0. Such a condition can not occur in a real-world vehicle collision. By definition, the coefficient of restitution must be less than 1.0 when residual crush occurs.

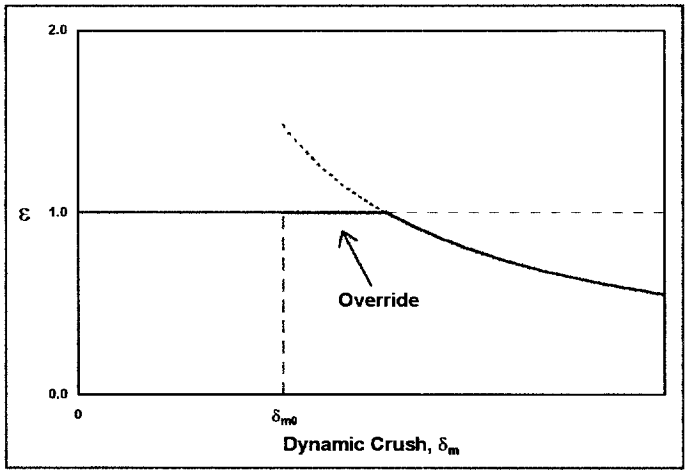

Figure 5: Restitution Equation Override For $K_1 < K_2$

These problems can be prevented by the use of realistic crush stiffness coefficients and restitution constants. A method is set forth in this paper for calculating the crush stiffness coefficients, K_1 and K_2, and the restitution constants, Γ and ρ, such that the above unrealistic characteristics are not specified for a vehicle.

METHOD

COMPREHENSIVE TESTS AVAILABLE

If complete and comprehensive tests are available for a vehicle, then these tests should be used to determine the stiffness coefficients and restitution constants. However, the results must be checked to insure compliance with the follow requirements:

A. The maximum value of the coefficient of restitution must be less than 1.0.

B. The coefficient of restitution must begin to decrease at, or before, the onset of residual crush.

Step 1

Determine the CRASH3 crush stiffness coefficients, A and B, using an appropriate method from the available test data [3].

Step 2

The next step is to determine the maximum magnitude for the coefficient of restitution and the value of the fitted restitution constant, Γ. From the crash test data, graph the coefficient of restitution versus residual crush. Create a best-fit line through the data points and extend this line to the zero residual crush level. The intercept with the y-axis, γ, will be the maximum value for the coefficient of restitution and should be less than 1.0.

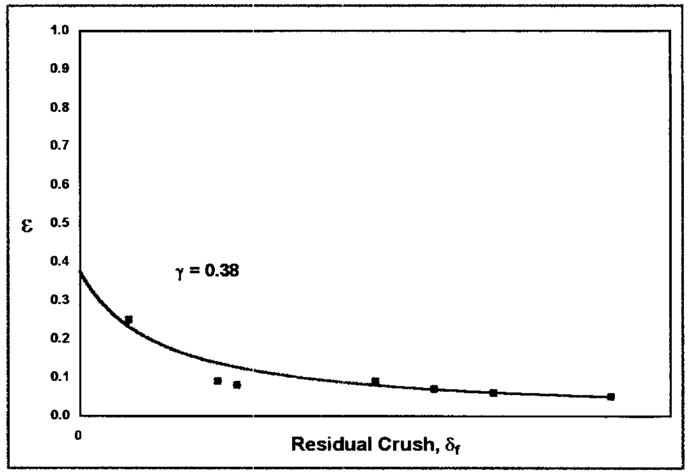

Figure 6: Coefficient of Restitution vs. Residual Crush

$$\varepsilon_{max} = \sqrt{\frac{K_2}{K_1}} = \gamma \qquad \text{Where } 0 < \gamma < 1.0 \tag{4}$$

Rearranging equation (4) results in an equation for K_2.

$$K_2 = \gamma^2 K_1 \tag{5}$$

The restitution constant, Γ, is defined as [1]:

$$\Gamma = \frac{A}{B}\sqrt{\frac{K_2}{K_1}} \tag{6}$$

Substituting equation (4) into equation (6) yields:

$$\Gamma = \gamma \frac{A}{B} \tag{7}$$

Step 3

The next step is to determine the value of K_1. To do this the relationship of residual crush to dynamic crush must be determined. Graph the measured values of residual crush versus dynamic crush.

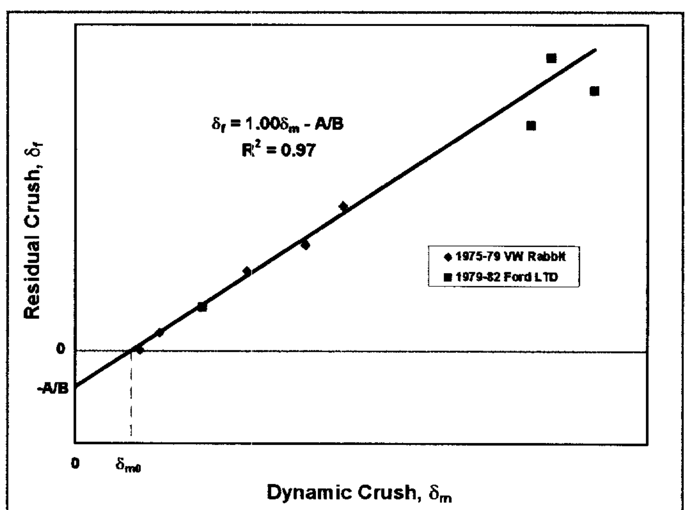

Figure 7: Residual vs. Dynamic Crush

The revised CRASH3/SMAC uses a simple slope-intercept equation (Eq. 8) to define the relationship between these variables.

$$\delta_f = \sqrt{\frac{K_1}{B}\delta_m - \frac{A}{B}} \qquad (8)$$

A best-fit line can be drawn from the y-axis intercept (-A/B) through the data points. The slope (β) of this best-fit line can be measured from the graph. Now the value of K_1 can be determined.

$$K_1 = \beta^2 B \qquad (9)$$

Step 4

Next the value of the fitted restitution constant, ρ, is determined. This is done by recognizing that the onset of residual crush (Eq. 10) must occur either before, or be coincident with, the onset of a decrease in the coefficient of restitution. The dynamic crush associated with the onset of residual crush is a function of the stiffness coefficients.

$$\delta_{m0} = \frac{A}{\sqrt{BK_1}} \qquad (10)$$

The dynamic crush associated with the maximum restitution is a function of the restitution constants and the magnitude of the coefficient of restitution at the onset of residual crush.

$$\delta_m = \frac{\Gamma}{\gamma - \rho} \qquad (11)$$

Set equation (10) equal to equation (11). This is done to meet the requirement that the coefficient of restitution must begin to decrease at, or before, the onset of residual crush.

$$\frac{A}{\sqrt{BK_1}} = \frac{\Gamma}{\gamma - \rho} \qquad (12)$$

Solving for ρ yields:

$$\rho = (1 - \beta)\gamma \qquad (13)$$

Step 5

The next step is to calculate K_2 using equation (5).

Step 6

In the final step, verify the results by graphing ε versus δ_f using equations (1) & (8) and the determined values for K_1, K_2, A, B, Γ and ρ. The results should match the test data graphed in step 2. Check that K_1 is not less than K_2.

COMPREHENSIVE TESTS NOT AVAILABLE

When comprehensive test data is not available, the revised CRASH3/SMAC stiffness coefficients and restitution constants can be estimated. Follow steps 1-6 with the following exceptions:

Step 2: Estimate γ using sound engineering judgement. The magnitude of γ should be less than 1.0.

Step 3: Estimate β using sound engineering judgement. The limited data available that has been plotted in figure 7 would indicate that a value of 1.0 for β is reasonable.

The determined coefficients/constants should be reviewed carefully to verify their accuracy and to insure that they represent realistic vehicle properties.

SUMMARY

1. The revised CRASH3 and SMAC would use identical crush stiffness coefficients and restitution constants to describe the crush response characteristics of vehicles.
2. The restitution portion of the force-deflection model has an underlying problem. The new force-deflection model coupled with improperly determined crush stiffness coefficients and restitution constants can create vehicle properties that are not realistic. Properly determined coefficients/constants, however, used in conjunction with the revised CRASH3/SMAC force-deflection model should result in improved accuracy over the original CRASH3 and SMAC programs when estimating collision severity.
3. A problem occurs when the programs set the magnitude of the coefficient of restitution equal to 1.0. In the real world, the coefficient of restitution may approach the theoretical magnitude of 1.0 but will never equal 1.0. A perfectly elastic vehicle structure simply does not exist in the real world.
4. A problem occurs when the loading stiffness coefficient, K_1, that is entered into the revised CRASH3/SMAC is less than the entered unloading stiffness coefficient, K_2. If $K_1 < K_2$, then in a full dimensional recovery collision, the absorbed energy would be less than the restored energy. This means that energy would be created in a collision. Therefore, it is not realistic for $K_1 < K_2$ when $\delta_f = 0$.
5. Another problem occurs when the entered values of $K_1 < K_2$. For partial dimensional recovery collisions, with K_1 less than K_2, a condition exists where residual crush is occurring and at the same time the coefficient of restitution is being set equal to 1.0. Such a condition can not occur in a real-world vehicle collision. By definition, the coefficient of restitution must be less than 1.0 when residual crush occurs.
6. A method for determining crush stiffness coefficients and restitution constants for the revised CRASH3 and SMAC has been set forth in this paper. Steps are taken in the method to prevent unrealistic modeling of vehicle properties. Finally, this method will provide a means to estimate the coefficients/constants in the absence of comprehensive testing.
7. The determined coefficients/constants should be reviewed carefully to verify their accuracy and to insure that they represent realistic vehicle properties.

REFERENCES

1. McHenry, Raymond R., and B. G. McHenry, "Effects of Restitution in the Application of Crush Coefficients," SAE Paper 970960, 1997.
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